

Name: _____

Use of books, notes or calculators is **NOT** permitted.

Please show all your work! Answers without appropriate supporting work may not receive full credit.

Clearly indicate your answers to each problem by underlining them or placing a box around your answers!

Trigonometric functions at the values $0, \pi/6, \pi/4, \pi/3, \pi/2$, etc must be evaluated!

T/F Questions are graded with NO PARTIAL CREDIT.

There is a total of 3 **DOUBLE-SIDED** pages to this exam including the cover page.

Exam Score

Problem	Score	Out of:
1		15
2		15
3		20
4		20
5		15
6		15
Total		100

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1. [15] For the True/False questions below, clearly circle your answer.

T or F If f is a function, and $f(x) = f(y)$, then $x = y$.

T or F A vertical line intersects the graph of a function at most once.

T or F $\lim_{x \rightarrow 4} \left(\frac{2x}{x-4} - \frac{8}{x-4} \right) = \lim_{x \rightarrow 4} \frac{2x}{x-4} - \lim_{x \rightarrow 4} \frac{8}{x-4}$.

T or F If the line $x = 0$ is a vertical asymptote of $y = f(x)$, then f is defined at 0.

T or F If $\lim_{x \rightarrow 5} f(x) = 2$ and $\lim_{x \rightarrow 5} g(x) = 0$, then $\lim_{x \rightarrow 5} f(x)/g(x)$ does not exist.

2. [15] Given the function $f(x) = \sqrt[3]{1 + e^x}$

(a) [5 pts] State the domain of f .

(b) [10 pts] Find f^{-1} , i.e. the inverse function of f .

3. [20] Compute the limits:

(a) [5 pts]

$$\lim_{x \rightarrow -1} \frac{x^4 - 3x + 5}{x^2 - 2}$$

(b) [5 pts]

$$\lim_{x \rightarrow -3^-} \frac{x - 1}{x^2(x + 3)}$$

(c) [5 pts]

$$\lim_{h \rightarrow 0} \frac{\sqrt{16 + h} - 4}{h}$$

(d) [5 pts]

$$\lim_{x \rightarrow -\infty} \frac{-x^3 + 2x^2 - 4}{2x^2 + x - 1}$$

4. [20] Let us consider the parametric curve given by

$$x(t) = \sqrt[3]{t} \quad y(t) = 1 - t \quad t \in \mathbb{R}.$$

(a) [5 pts] Eliminate the parameter to find the Cartesian equation of the curve.

(b) [5 pts] Identify the curve.

(c) [5 pts] Graph the curve for $t \in [-1, 1]$. Identify the starting and the ending points and draw an arrow on the curve to show the direction of increasing t .

(d) [5 pts] The Cartesian equation of the curve you obtained at point a) gives y as function of x . Is this function one-to-one, on the interval $[-1, 1]$? Why?

5. [15] Find the equation of the tangent line to the graph of $y = x^2 + 3x + 2$ at the point $(1, 6)$. Use the limit definition to find the slope of the tangent line.

6. [15] Suppose that

$$f(x) = \begin{cases} \frac{a}{x^2} + 1 & \text{if } 0 < x < 1, \\ x + 2 & \text{if } 1 \leq x \leq 2, \\ bx^2 & \text{if } 2 < x \end{cases}$$

Find a and b such that f will be continuous at both $x = 1$ and $x = 2$.