

Name: _____

Use of books, notes or calculators is **NOT** permitted.

Please show all your work! Answers without appropriate supporting work may not receive full credit.

Clearly indicate your answers to each problem by underlining them or placing a box around your answers!

Trigonometric functions at the values $0, \pi/6, \pi/4, \pi/3, \pi/2$, etc must be evaluated!

T/F Questions are graded with NO PARTIAL CREDIT.

There is a total of 3 **DOUBLE-SIDED** pages to this exam including the cover page.

Exam Score

Problem	Score	Out of:
1		15
2		15
3		20
4		20
5		15
6		15
Total		100

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1. [15] For the True/False questions below, clearly circle your answer.

T or F If f is a function, then $f(x + y) = f(x) + f(y)$.

T or F If $\lim_{x \rightarrow 5} f(x) = 0$ and $\lim_{x \rightarrow 5} g(x) = 0$, then $\lim_{x \rightarrow 5} f(x)/g(x)$ does not exist.

T or F $\lim_{x \rightarrow 2} \frac{x^2+6x-7}{x^2+5x-6} = \frac{\lim_{x \rightarrow 2}(x^2+6x-7)}{\lim_{x \rightarrow 2}(x^2+5x-6)}$.

T or F If the line $x = 1$ is a vertical asymptote of $y = f(x)$, then f is not defined at 1.

T or F If f is a continuous function, and $f(1) = 1$ and $f(3) = -2$, then there exists a number c in $(1, 3)$ such that $f(c) = 0$.

2. [15] Given the function $f(x) = \sqrt{1 - e^x}$

(a) [5 pts] State the domain of f .

(b) [10 pts] Find f^{-1} , i.e. the inverse function of f .

3. [20] Compute the limits:

(a) [5 pts]

$$\lim_{x \rightarrow -2} \frac{x^3 - 3x + 5}{x + 3}$$

(b) [5 pts]

$$\lim_{x \rightarrow -1^-} \frac{x - 1}{x^3(x + 1)}$$

(c) [5 pts]

$$\lim_{h \rightarrow 0} \frac{(2 + h)^2 - 4}{h}$$

(d) [5 pts]

$$\lim_{x \rightarrow \infty} \frac{-x^2 + x - 4}{2x^2 + 3x - 1}$$

4. [20] Let us consider the parametric curve given by

$$x(t) = \sqrt{t} \quad y(t) = 1 - t \quad t \in [0, +\infty).$$

(a) [5 pts] Eliminate the parameter to find the Cartesian equation of the curve.

(b) [5 pts] Identify the curve.

(c) [5 pts] Graph the curve for $t \in [0, 2]$. Identify the starting and the ending points and draw an arrow on the curve to show the direction of increasing t .

(d) [5 pts] The Cartesian equation of the curve you obtained at point a) gives y as function of x . Is this function one-to-one on the interval $[0, 2]$? Why?

5. [15] Find the equation of the tangent line to the graph of $y = x^2 - x + 1$ at the point $(2, 3)$. Use the limit definition to find the slope of the tangent line.

6. [15] Suppose that

$$f(x) = \begin{cases} \frac{5}{x} & \text{if } 0 < x < 1, \\ ax + 2 & \text{if } 1 \leq x \leq 2, \\ bx^2 & \text{if } 2 < x \end{cases}$$

Find a and b such that f will be continuous at both $x = 1$ and $x = 2$.