Name: ____

Use of books, notes or calculators is **NOT** permitted.

Please show all your work! Answers without appropriate supporting work my not receive full credit.

Clearly indicate your answers to each problem by underlining them or placing a $\boxed{\text{box}}$ around your answers!

Trigonometric functions at the values $0, \pi/6, \pi/4, \pi/3, \pi/2$, etc must be evaluated!

T/F Questions are graded with NO PARTIAL CREDIT.

Exam Score

Problem	Score	Out of:
1		10
2		20
3		10
4		25
5		15
6		20
Total		100

1. [**10**] For the True/False questions below, clearly circle your answer.

T or F If f and g are continuous on $[a,b]$, then $\int_a^b [f(x)+g(x)]dx = \int_a^b f(x)dx + \int_a^b g(x)dx$

T or F If f and g are continuous on $[a,b]$, then $\int_a^b [f(x)g(x)]dx = \int_a^b f(x)dx \cdot \int_a^b g(x)dx$

T or F If $p(x)$ is a polynomial function, then p has exactly one antiderivative whose graph contains the origin.

T or F If $\int_{-a}^{a} f(x)\,dx = 0$, then $f(x) = 0$ for all x in the interval $[-a,a]$.

T or F $\int_1^2 \frac{1}{x}\,dx = \ln|x| \big|_{x=1}^{x=2} = \ln 2 - \ln 1 = \ln 2.$

2. [**20**] Find the function $f(t)$, if $f''(t) = 24t^2 + 6t + 10$, $f'(0) = -3$ and $f(-1) = 5$.

$$\left.\begin{array}{l} f'(t) = 8t^3 + 3t^2 + 10t + C \\ f'(0) = -3 \end{array}\right\} \Rightarrow \boxed{C = -3}$$

$$f'(t) = 8t^3 + 3t^2 + 10t - 3$$

$$f(t) = 2t^4 + t^3 + 5t^2 - 3t + D$$

$$\left.\begin{array}{l} f(-1) = 2 - 1 + 5 + 3 + D = 9 + D \\ f(-1) = 5 \end{array}\right\} \quad 9 + D = 5 \Rightarrow \boxed{D = -4}$$

3. [**10**] Explain why $\int_{-\pi/2}^{\pi/2} \sin^5 x \cos^2 x\,dx$ is 0.

$\sin^5 x \cos^2 x$ is an odd function. So $\int_{-\pi/2}^{\pi/2}$ odd fcn $= 0$

$\sin x$ odd $\Rightarrow$ $\sin^5 x$ is odd $\}\Rightarrow$ their product is odd fcn

4. [**25**] Evaluate the integrals

(a) [**6**] $\int xe^{x^2}\,dx$ $= \int e^u \cdot \frac{1}{2}\,du = \frac{1}{2}\int e^u\,du = \frac{1}{2}e^u + C$

$u = x^2$ $du = 2x\,dx$ $x\,dx = \frac{1}{2}\,du$ $= \frac{1}{2}e^{x^2} + C$

(b) [**6**] $\int_0^{\sqrt{\pi}} t\cos t^2\,dt$ $= \int_0^{\pi} \cos u \cdot \frac{1}{2}\,du =$

$u = t^2$ $du = 2t\,dt$

$= \frac{1}{2}\int_0^{\pi} \cos u\,du =$

$t = 0$ $u = 0$

$t = \sqrt{\pi}$ $u = \pi$

$= \frac{1}{2}\sin u \Big|_0^{\pi} = \frac{1}{2}(\sin \pi - \sin 0) = 0$

(c) [**6**] $\int \ln t\,dt$ $= t\ln t - \int t \cdot \frac{1}{t}\,dt = t\ln t - t + C$

(d) [**7**] $\int_1^2 -\frac{\ln x}{x^2}\,dx$ $= \int_1^2 \left(-\frac{1}{x^2}\right)\ln x\,dx = \frac{1}{x}\ln x\Big|_1^2 - \int_1^2 \frac{1}{x}\cdot\frac{1}{x}\,dx$

$f' = -\frac{1}{x^2}$ $f = \frac{1}{x}$

$= \frac{1}{2}\ln 2 + \int_1^2 -\frac{1}{x^2}\,dx$

$g = \ln x$ $g' = \frac{1}{x}$

$= \frac{1}{2}\ln 2 + \frac{1}{x}\Big|_1^2$

$= \frac{1}{2}\ln 2 + \frac{1}{2} - 1$

5. [**15**] If

$$F(x) = \int_0^{x^3} \sin(t^2+1)\,dt$$

find $F'(x)$.

Let $H(t)$ be an antiderivative of $\sin(t^2+1)$ i.e. $H'(t) = \sin(t^2+1)$

Then by the Fund Th of Calculus

$$F(x) = \int_0^{x^3} \sin(t^2+1)\,dt = H(x^3) - H(0)$$

So $F'(x) = 3x^2 H'(x^3) - 0 = 3x^2 \sin((x^3)^2+1) = 3x^2 \sin(x^6+1)$

6. [**20**] Evaluate the integral

$$\int \cos^2(x)\sin^2(x)\,dx$$

$$\cos^2 x = \frac{1+\cos 2x}{2} \qquad \sin^2 x = \frac{1-\cos 2x}{2}$$

$$\int \frac{1+\cos 2x}{2} \cdot \frac{1-\cos 2x}{2}\,dx = \int \frac{1-\cos^2 2x}{4}\,dx =$$

$$= \frac{1}{4}x - \frac{1}{4}\int \cos^2 2x\,dx = \frac{1}{4}x - \frac{1}{4}\int \frac{1+\cos 4x}{2}\,dx =$$

$$= \frac{1}{4}x - \frac{1}{4}\int \frac{1}{2}dx - \frac{1}{4}\cdot\frac{1}{2}\int \cos 4x\,dx$$

$$= \frac{1}{4}x - \frac{1}{8}x - \frac{1}{8}\cdot\frac{1}{4}\sin 4x + C$$

$$= \frac{1}{8}x - \frac{1}{32}\sin 4x + C$$