Name: ______________________________

Use of books, notes or calculators is **NOT** permitted.

Please show all your work! Answers without appropriate supporting work may not receive full credit.

Clearly indicate your answers to each problem by underlining them or placing a $\boxed{\text{box}}$ around your answers!

Trigonometric functions at the values $0, \pi/6, \pi/4, \pi/3, \pi/2$, etc must be evaluated!

T/F Questions are graded with NO PARTIAL CREDIT.

There is a total of 4 **DOUBLE-SIDED** pages to this exam including the cover page.

Solutions to Test 2A

Exam Score

Problem	Score	Out of:
1		10
2		10
3		20
4		20
5		25
6		10
7		10
Total		105

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1. [**10**] For the True/False questions below, clearly circle your answer.

T or F If f is continuous at a, then f is differentiable at a.
T or F $\frac{d}{dx}\ln\pi = \frac{1}{\pi}$.
T or F If $f(x) = x^4$, then $\lim_{x\to 2}\frac{f(x)-f(2)}{x-2} = 32$.
T or F If f is differentiable, and $f'' > 0$ on (a,b), then f is concave upwards on (a,b).
T or F If f and g are differentiable, then $\frac{d}{dx}[f(x)g(x)] = \frac{d}{dx}f(x)\cdot\frac{d}{dx}g(x)$.

2. [**10**] Use the limit definition of derivative to find the derivative of $f(x) = x^2 + x$.

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{(x+h)^2+(x+h)-(x^2+x)}{h} =$$

$$= \lim_{h\to 0}\frac{\cancel{x^2}+2xh+h^2+\cancel{x}+h-\cancel{x^2}-\cancel{x}}{h} =$$

$$= \lim_{h\to 0}\frac{2xh+h^2+h}{h} = \lim_{h\to 0}(2x+h+1)$$

$$= 2x+1$$

So: $\boxed{f'(x) = 2x+1}$

3. [**20**]Find the indicated derivatives.

(a) [5 pts] $f(x) = x^3e^{x^2}$. Find $f'(1)$

$f'(x) = 3x^2e^{x^2} + x^3 \cdot e^{x^2} \cdot 2x$

$$\boxed{f'(1) = 3e + 2e = 5e}$$

(b) [5 pts] $g(y) = \frac{1}{\sqrt[3]{y^2}} + \cos 2\pi y + \ln y^2$, find $\frac{dg}{dy}$

$g(y) = y^{-\frac{2}{3}} + \cos 2\pi y + 2\ln y$

$$\boxed{g'(y) = -\frac{2}{3}y^{-\frac{5}{3}} - 2\pi \sin 2\pi y + \frac{2}{y}}$$

(c) [5 pts] $h(t) = \tan(t)\arcsin(t^2)$. Find $h'(t)$

$h'(t) = \sec^2(t)\arcsin(t^2) + \tan(t) \cdot \frac{1}{\sqrt{1-(t^2)^2}} \cdot 2t$

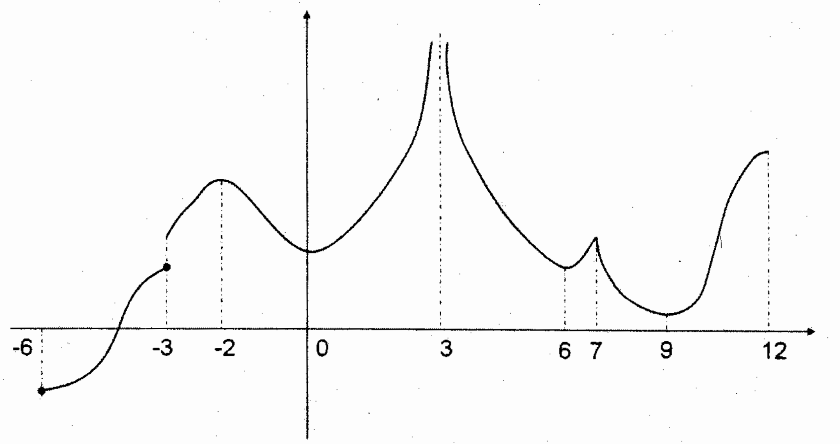
$$\boxed{h'(t) = \sec^2(t)\arcsin(t^2) + \frac{2t\tan(t)}{\sqrt{1-t^4}}}$$

(d) [5 pts] $r(s) = \frac{s^3}{1-s^2}$. Find $\frac{dr}{ds}$

$\frac{dr}{ds} = \frac{3s^2(1-s^2) - s^3 \cdot (-2s)}{(1-s^2)^2} = \frac{3s^2 - 3s^4 + 2s^4}{(1-s^2)^2} =$

$$\boxed{= \frac{3s^2 - s^4}{(1-s^2)^2}}$$

4. [**20**] The graph of the function $f : [-6, 12) \to \mathbf{R}$ is shown below.



(a) [5 pts] State with reasons the points at which f is not differentiable.

-3 (f is not even continuous at this point)

3 (f is not even continuous at this point)

7 (f' does not exist. $f'_\ell(7) \neq f'_r(7)$)

(b) [5 pts] State with reasons the intervals where $f' > 0$.

$(-6,-3)$
$(-3,-2)$
$(0,3)$
$(6,7)$
$(9,12)$

because f is increasing on these intervals.

(c) [5 pts] State with reasons the intervals where $f' < 0$.

$(-2,0)$
$(3,6)$
$(7,9)$

because f is decreasing on these intervals.

(d) [5 pts] State with reasons the points where $f' = 0$.

-2
0
6
9

because the tangent line is horizontal at these points.

5. [**25**] Let us consider the function $f(x) = x^3 + 3x^2 - 9x - 6$.

(a) [10 pts] On what interval is f increasing?

For a differentiable function, f is increasing if and only if $f'(x) > 0$

$$f'(x) = 3x^2 + 6x - 9 = 3(x^2 + 2x - 3) = 3(x+3)(x-1)$$

$$f'(x) > 0 \iff 3(x+3)(x-1) > 0$$

x	$-\infty$		-3		1		$+\infty$
$(x+3)$		$- - -$	0	$+ + + +$		$+ + + +$	
$(x-1)$		$- - - -$		$- -$	0	$+ + + + +$	
$(x+3)(x-1)$		$+ + +$	0	$- -$	0	$+ + + + +$	

So $f'(x) > 0$ for

$$\boxed{x \in (-\infty, -3) \cup (1, +\infty)}$$

(b) [10 pts] On what interval is f concave upward?

For a diff'b fcn, f is concave upward if and only if $f''(x) > 0$

$$f''(x) = (3x^2 + 6x - 9)' = 6x + 6 = 6(x+1)$$

$$f''(x) > 0 \iff 6(x+1) > 0 \iff x > -1$$

$$\boxed{\text{So } f \text{ is concave upwards } (f''(x) > 0) \text{ for } x > -1 \text{ or } x \in (-1, +\infty)}$$

(c) [**5 Extra Bonus** pts] Show that f has a solution in the interval $(-1, 0)$.

f is a <u>continuous fcn</u> on the interval $[-1, 0]$ (Why? Because is a polynomial)

$$f(-1) = (-1)^3 + 3(-1)^2 - 9(-1) - 6 = -1 + 3 + 9 - 6 = 5$$

$$f(0) = -6$$

$f(-1)f(0) < 0 \Rightarrow$ According to the <u>Intermediate Value Theorem</u> the result follows.

6. [**10**] Use implicit differentiation to find an equation of the tangent line to the ellipse $x^2 + xy + y^2 = 3$ at the point $(1, 1)$.

Eq of tangent line: $y - 1 = m(x-1)$ $m = ?$

$$2x + y + x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(x+2y) = -(2x+y) \qquad \frac{dy}{dx} = -\frac{2x+y}{x+2y}$$

@ $(1,1)$ $m = \frac{dy}{dx} = -\frac{2\cdot 1+1}{1+2\cdot 1} = -\frac{3}{3} = -1$

So: eq of tang line is $y - 1 = (-1)(x-1)$ or $y = -x + 2$

7. [**10**] Use logarithmic differentiation to find the derivative of the function $y = x^{\cos x}$.

$$y = x^{\cos x}$$

$$\ln y = \ln x^{\cos x} = \cos x \cdot \ln x \quad \Big| \frac{d}{dx}$$

$$\frac{y'}{y} = -\sin x \cdot \ln x + \cos x \cdot \frac{1}{x}$$

$$\boxed{y' = x^{\cos x}\left(-\sin x \cdot \ln x + \frac{\cos x}{x}\right)}$$