Name: ______________________________

Use of books, notes or calculators is **NOT** permitted.

Please show all your work! Answers without appropriate supporting work my not receive full credit.

Clearly indicate your answers to each problem by underlining them or placing a $\boxed{\text{box}}$ around your answers!

Trigonometric functions at the values $0, \pi/6, \pi/4, \pi/3, \pi/2$, etc must be evaluated!

T/F Questions are graded with NO PARTIAL CREDIT.

There is a total of 3 **DOUBLE-SIDED** pages to this exam including the cover page.

Solutions to Test 1B

Exam Score

Problem	Score	Out of:
1		15
2		15
3		20
4		20
5		15
6		15
Total		100

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1. [**15**] For the True/False questions below, clearly circle your answer.

T or F If f is a function, and $f(x) = f(y)$, then $x = y$.

T or F A vertical line intersects the graph of a function at most once.

T or F $\lim_{x\to 4}\left(\frac{2x}{x-4} - \frac{8}{x-4}\right) = \lim_{x\to 4}\frac{2x}{x-4} - \lim_{x\to 4}\frac{2x}{x-4}$.

T or F If the line $x = 0$ is a vertical asymptote of $y = f(x)$, then f is defined at 0.

T or F If $\lim_{x\to 5} f(x) = 2$ and $\lim_{x\to 5} g(x) = 0$, then $\lim_{x\to 5} f(x)/g(x)$ does not exist.

2. [**15**] Given the function $f(x) = \sqrt[3]{1 + e^x}$

(a) [5 pts] State the domain of f.

$\sqrt[3]{\ }$ is defined for all reals, so there are no restrictions for $1+e^x$. Consequently

$$\boxed{D = \mathbb{R}}$$

(b) [10 pts] Find f^{-1}, i.e. the inverse function of f.

$$y = \sqrt[3]{1+e^x} \Rightarrow y^3 = 1 + e^x \Rightarrow e^x = y^3 - 1 \Rightarrow$$
$$x = \ln(y^3 - 1)$$

$$\boxed{f^{-1}(x) = \ln(x^3 - 1)}$$

3. [**20**] Compute the limits:

(a) [5 pts]

$$\lim_{x\to -1}\frac{x^4-3x+5}{x^2-2} = \frac{(-1)^4-3(-1)+5}{(-1)^2-2} = \frac{1+3+5}{1-2} =$$

$$= \frac{9}{-1} = \boxed{-9}$$

(b) [5 pts]

$$\lim_{x\to -3^-}\frac{x-1}{x^2(x+3)} = \frac{-3-1}{(-3)^2\,(-3^-+3)} =$$

$$= \frac{-4}{9\cdot(-0)} = \boxed{+\infty}$$

(c) [5 pts]

$$\lim_{h\to 0}\frac{\sqrt{16+h}-4}{h} = \left[\frac{0}{0}\right]$$

$$\lim_{h\to 0}\frac{\sqrt{16+h}-4}{h} = \lim_{h\to 0}\frac{\cancel{16}+\cancel{h}-\cancel{16}}{\cancel{h}\left[\sqrt{16+h}+4\right]} = \lim_{h\to 0}\frac{1}{\sqrt{16+h}+4} = \boxed{\frac{1}{8}}$$

(d) [5 pts]

$$\lim_{x\to -\infty}\frac{-x^3+2x^2-4}{2x^2+x-1} = \lim_{x\to -\infty}\frac{-x^{\cancel{3}}\left[1-\frac{2}{x}+\frac{4}{x^3}\right]}{\cancel{x^2}\left[2+\frac{1}{x}-\frac{1}{x^2}\right]}$$

$$= \lim_{x\to -\infty}(-x)\cdot\lim_{x\to -\infty}\frac{1-\frac{2}{x}+\frac{4}{x^3}}{2+\frac{1}{x}-\frac{1}{x^2}} = \lim_{x\to -\infty}(-x)\cdot\lim_{x\to -\infty}\frac{1-0+0}{2+0-0}$$

$$= +\infty\cdot\frac{1}{2} = \boxed{+\infty}$$

4. [**20**] Let us consider the parametric curve given by

$$x(t) = \sqrt[3]{t} \qquad y(t) = 1 - t \qquad t \in \mathbb{R}.$$

(a) [5 pts] Eliminate the parameter to find the Cartesian equation of the curve.

$x = \sqrt[3]{t} \Rightarrow x^3 = t$ Substitute this in y's equation

$y = 1 - t = 1 - x^3$

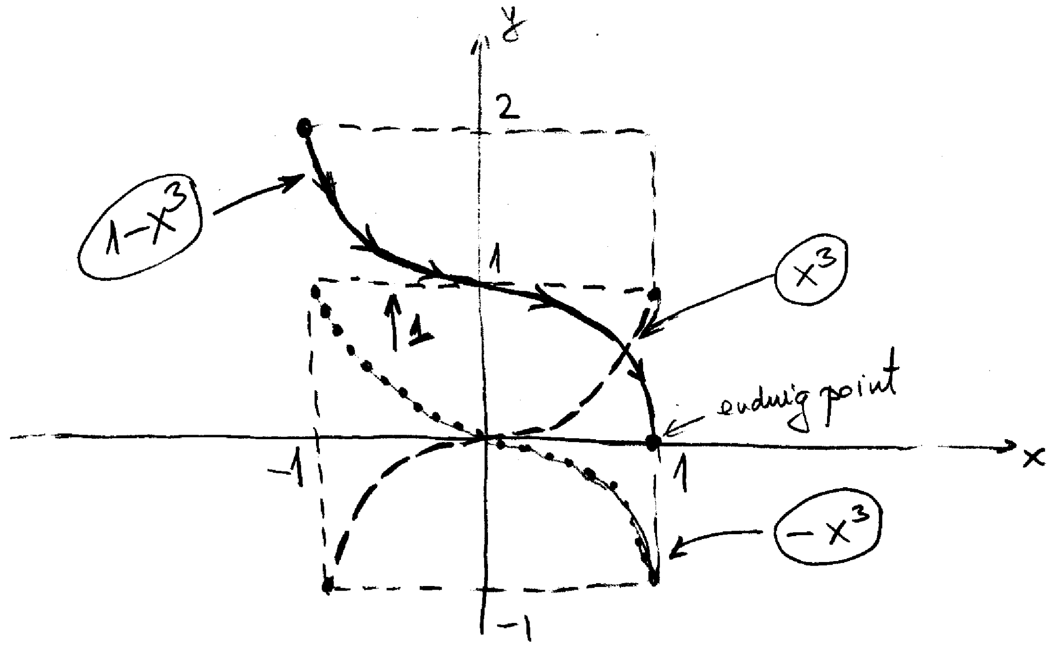
$\boxed{y = 1 - x^3}$

(b) [5 pts] Identify the curve.

$y = x^3$ is a $\boxed{\text{CUBIC PARABOLA}}$, $y = 1 - x^3$ is obtained from $y = x^3$ by a fliping around x-axis and 1 unit shift up. So is still a CUBIC PARABOLA.

(c) [5 pts] Graph the curve for $t \in [-1, 1]$. Identify the starting and the ending points and draw an arrow on the curve to show the direction of increasing t.

t	x	y
-1	-1	2
0	0	1
1	1	0



(d) [5 pts] The Cartesian equation of the curve you obtained at point a) gives y as function of x. Is this function one-to-one, on the interval $[-1, 1]$? Why?

$y = 1 - x^3$ and the graph of this fcn is shown above.

Any horizontal line intersects the graph of $y = 1 - x^3$ in <u>exactly one point</u>! According to the <u>horizontal line test</u> $y = 1 - x^3$ is <u>one-to-one</u>.

5. [**15**] Find the equation of the tangent line to the graph of $y = x^2 + 3x + 2$ at the point $(1, 6)$. Use the limit definition to find the slope of the tangent line.

Eq of tangent line is $y - 6 = m(x-1)$

We need only to evaluate the slope, i.e,

$$m = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0} \frac{(1+h)^2 + 3(1+h) + 2 - 6}{h} =$$

$$= \lim_{h \to 0} \frac{\cancel{1} + 2h + h^2 + \cancel{3} + 3h + \cancel{2} - \cancel{6}}{h} = \lim_{h \to 0} \frac{5h + h^2}{h} =$$

$$= \lim_{h \to 0} (5 + h) = 5.$$

So eq of tangent line is $\boxed{y - 6 = 5(x-1)}$

or

$\boxed{y = 5x + 1}$

6. [**15**] Suppose that

$$f(x) = \begin{cases} \frac{a}{x^2} + 1 & \text{if } 0 < x < 1, \\ x + 2 & \text{if } 1 \le x \le 2, \\ bx^2 & \text{if } 2 < x \end{cases}$$

Find a and b such that f will be continuous at both $x = 1$ and $x = 2$.

f is continuous @ 1 if and only if $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$

$$\left.\begin{aligned} &\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} \left(\frac{a}{x^2} + 1\right) = a + 1 \\ &\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (x + 2) = 3 \\ &f(1) = 3 \end{aligned}\right\} \Rightarrow$$

For f to be continuous @ 1 we need to have $a + 1 = 3$ that is $\boxed{a = 2}$

Similarly f is continuous @ 2 if $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = f(2)$

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (x + 2) = 4$$

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} bx^2 = 4b$$

$$f(2) = 4$$

$4 = 4b$ that is

$\boxed{b = 1}$