Name: ______________________________

Use of books, notes or calculators is **NOT** permitted.

Please show all your work! Answers without appropriate supporting work my not receive full credit.

Clearly indicate your answers to each problem by underlining them or placing a $\boxed{\text{box}}$ around your answers!

Trigonometric functions at the values $0, \pi/6, \pi/4, \pi/3, \pi/2$, etc must be evaluated!

T/F Questions are graded with NO PARTIAL CREDIT.

There is a total of 3 **DOUBLE-SIDED** pages to this exam including the cover page.

Solutions to Test 1A

Exam Score

Problem	Score	Out of:
1		15
2		15
3		20
4		20
5		15
6		15
Total		100

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1. [**15**] For the True/False questions below, clearly circle your answer.

   T or F If f is a function, then $f(x+y)=f(x)+f(y)$.

   T or F If $\lim_{x\to 5} f(x)=0$ and $\lim_{x\to 5} g(x)=0$, then $\lim_{x\to 5} f(x)/g(x)$ does not exist.

   T or F $\lim_{x\to 2} \frac{x^2+6x-7}{x^2+5x-6} = \frac{\lim_{x\to 2}(x^2+6x-7)}{\lim_{x\to 2}(x^2+5x-6)}$.

   T or F If the line $x=1$ is a vertical asymptote of $y=f(x)$, then f is not defined at 1.

   T or F If f is a continuous function, and $f(1)=1$ and $f(3)=-2$, then there exists a number c in $(1,3)$ such that $f(c)=0$.

2. [**15**] Given the function $f(x)=\sqrt{1-e^x}$

   (a) [5 pts] State the domain of f.

   $\sqrt{\ }$ is defined only for nonnegative numbers, so we need to impose $1-e^x \geq 0 \iff e^x \leq 1 \iff x \leq \ln 1 \iff$

   $x \leq 0$

   $$\boxed{D=(-\infty,0] \text{ or } D=\{x \,/\, x\leq 0\}}$$

   (b) [10 pts] Find f^{-1}, i.e. the inverse function of f.

   $y=\sqrt{1-e^x} \Rightarrow y^2=1-e^x \Rightarrow e^x=1-y^2$

   $x=\ln(1-y^2)$

   $$\boxed{f^{-1}(x)=\ln(1-x^2)}$$

3. [**20**] Compute the limits:

(a) [5 pts]

$$\lim_{x\to -2} \frac{x^3-3x+5}{x+3} = \frac{(-2)^3-3(-2)+5}{-2+3} = \frac{-8+6+5}{1} = \boxed{3}$$

(b) [5 pts]

$$\lim_{x\to -1^-} \frac{x-1}{x^3(x+1)} = \frac{-1-1}{(-1)^3(-1^-+1)} = \frac{-2}{(-1)(-0)} =$$

$$= \frac{2}{-0} = \boxed{-\infty}$$

(c) [5 pts]

$$\lim_{h\to 0} \frac{(2+h)^2-4}{h} = \lim_{h\to 0} \frac{\cancel{4}+4h+h^2-\cancel{4}}{h} =$$

$$= \lim_{h\to 0} \frac{4h+h^2}{h} = \lim_{h\to 0} (4+h) = \boxed{4}$$

(d) [5 pts]

$$\lim_{x\to\infty} \frac{-x^2+x-4}{2x^2+3x-1} = \lim_{x\to\infty} \frac{\cancel{x^2}\left(-1+\frac{1}{x}-\frac{4}{x^2}\right)}{\cancel{x^2}\left(2+\frac{3}{x}-\frac{1}{x^2}\right)} =$$

$$= \lim_{x\to\infty} \frac{-1+\overset{\to 0}{\frac{1}{x}}-\overset{\to 0}{\frac{4}{x^2}}}{2+\underset{\to 0}{\frac{3}{x}}-\underset{\to 0}{\frac{1}{x^2}}} = \boxed{-\frac{1}{2}}$$

4. [**20**] Let us consider the parametric curve given by

$$x(t) = \sqrt{t} \qquad y(t) = 1 - t \qquad t \in [0, +\infty).$$

(a) [5 pts] Eliminate the parameter to find the Cartesian equation of the curve.

$x = \sqrt{t} \Rightarrow x^2 = t$ Substitute this in y's equation

$y = 1 - t = 1 - x^2$

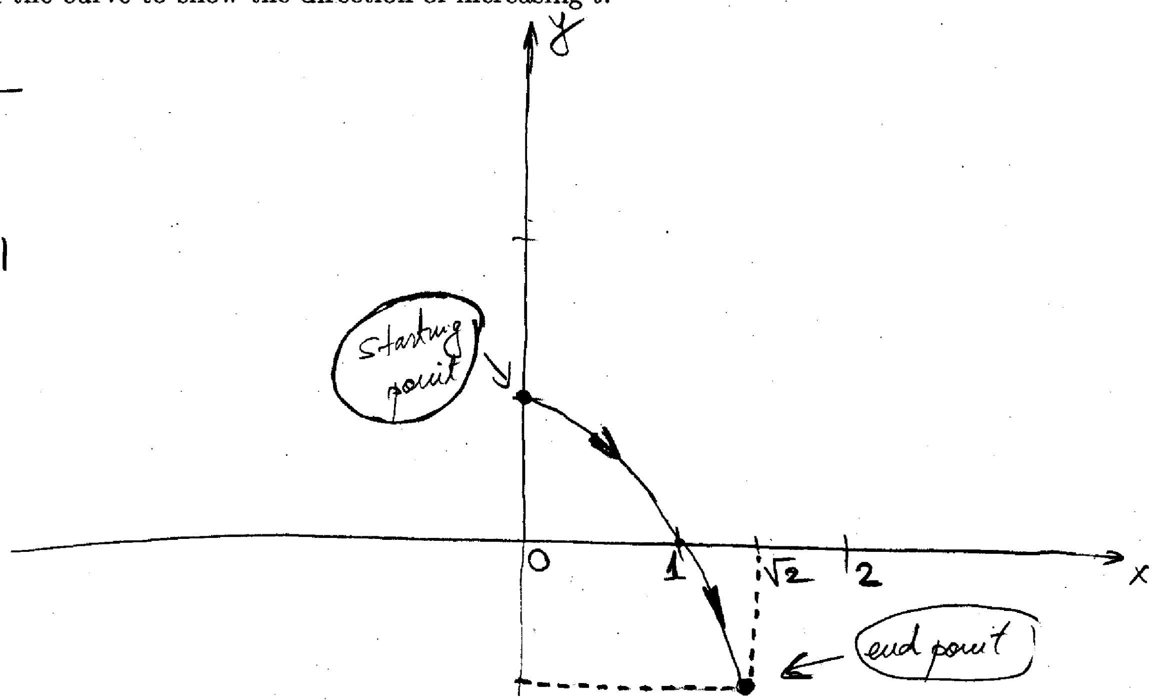
$\boxed{y = 1 - x^2}$

(b) [5 pts] Identify the curve. Parabola

$y = x^2$ is a parabola $\rightarrow$ $y = -x^2$ is a fliped parabola around x-axis $\rightarrow$ $y = 1 - x^2$ is 1 unit shifted up so still a parabola

(c) [5 pts] Graph the curve for $t \in [0, 2]$. Identify the starting and the ending points and draw an arrow on the curve to show the direction of increasing t.

t	x	y
0	0	1
1	1	0
2	$\sqrt{2}$	-1



(d) [5 pts] The Cartesian equation of the curve you obtained at point a) gives y as function of x. Is this function one-to-one on the interval $[0, 2]$? Why?

$y = 1 - x^2$ Any horizontal line intersects the graph of $y = 1 - x^2$ in exactly one point According to the horizontal line test

$y = 1 - x^2$ is one to one

5. [**15**] Find the equation of the tangent line to the graph of $y = x^2 - x + 1$ at the point $(2, 3)$. Use the limit definition to find the slope of the tangent line.

eq of tang line $y - 3 = m(x-2)$

We need only to evaluate the slope i.e

$$m = \lim_{h\to 0} \frac{f(2+h) - f(2)}{h} = \lim_{h\to 0} \frac{(2+h)^2 - (2+h) + 1 - 3}{h} =$$

$$= \lim_{h\to 0} \frac{\cancel{4} + 4h + h^2 - \cancel{2} - h + \cancel{1} - \cancel{3}}{h} = \lim_{h\to 0} \frac{3h + h^2}{h} =$$

$$= \lim_{h\to 0} [3 + h] = 3$$

So equation of the tangent line is $\boxed{y - 3 = 3(x-2)}$

or $\boxed{y = 3x - 3}$

6. [**15**] Suppose that

$$f(x) = \begin{cases} \frac{5}{x} & \text{if } 0 < x < 1, \\ ax + 2 & \text{if } 1 \le x \le 2, \\ bx^2 & \text{if } 2 < x \end{cases}$$

Find a and b such that f will be continuous at both $x = 1$ and $x = 2$.

f is continuous @ 1 if and only if $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(1)$

$$\left.\begin{aligned} &\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} \frac{5}{x} = 5 \\ &\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (ax+2) = a+2 \\ &f(1) = a+2 \end{aligned}\right\} \Rightarrow$$

f is cont at 1 if $a + 2 = 5$ that is $\boxed{a = 3}$

f is continuous @ 2 if and only if $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2)$

$$\left.\begin{aligned} &\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (ax+2) = 2a+2 = 8 \\ &\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} bx^2 = 4b \\ &f(2) = 4b \end{aligned}\right\} \Rightarrow$$

f is cont @ 2 if $4b = 8$ that is $\boxed{b = 2}$