Name: ______________________________

Use of books, notes or calculators is **NOT** permitted.

Please show all your work! Answers without appropriate supporting work my not receive full credit.

Clearly indicate your answers to each problem by underlining them or placing a box around your answers!

Trigonometric functions at the values $0, \pi/6, \pi/4, \pi/3, \pi/2$, etc must be evaluated!

T/F Questions are graded with NO PARTIAL CREDIT.

Solutions to Practice Test 1

Exam Score

Problem	Score	Out of:
1		15
2		20
3		15
4		15
5		20
6		8
7		7
Total		100

1. [**15**] For the True/False questions below, clearly circle your answer.

   T or F If f and g are functions, then $f \circ g = g \circ f$.

   T or F If p is a polynomial then $\lim_{x \to a} = p(a)$.

   T or F $\lim_{x\to 1} \frac{x^2+6x-7}{x^2+5x-6} = \frac{\lim_{x\to 1}(x^2+6x-7)}{\lim_{x\to 1}(x^2+5x-6)}$.

   T or F A function can have two different horizontal asymptotes.

   T or F If $f(1) = -3$ and $f(2) = 2$, then there exists a number c in $(1, 2)$ such that $f(c) = 0$.

2. [**20**] Compute the limits:

   (a) [5 pts]

$$\lim_{x\to -1} \frac{x^2+3x+2}{x+2} = \frac{(-1)^2+3(-1)+2}{-1+2} = \frac{0}{1} = \boxed{0}$$

   (b) [5 pts]

$$\lim_{x\to -2^+} \frac{x-1}{x^2(x+2)} = \frac{-2-1}{4(+0)} = \frac{-3}{+0} = \boxed{-\infty}$$

   (c) [5 pts]

$$\lim_{t\to 0} \frac{\sqrt{t^2+4}-2}{t^2} = \lim_{t\to 0} \frac{\cancel{t^2}+\cancel{4}-\cancel{4}}{\cancel{t^2}(\sqrt{t^2+4}+2)} = \lim_{t\to 0} \frac{1}{\sqrt{t^2+4}+2}$$

$$= \frac{1}{\sqrt{4}+2} = \boxed{\frac{1}{4}}$$

   (d) [5 pts]

$$\lim_{x\to\infty} \frac{3x^2-x+5}{x^3+x-1} = \lim_{x\to\infty} \frac{x^2\left(3-\frac{1}{x}+\frac{5}{x^2}\right)}{x^3\left(1+\frac{1}{x^2}-\frac{1}{x^3}\right)} =$$

$$= \lim_{x\to\infty} \frac{3-\frac{1}{x}+\frac{5}{x^2}}{x\left(1+\frac{1}{x^2}-\frac{1}{x^3}\right)} = \frac{3-0+0}{\infty(1+0-0)} = \frac{3}{\infty} = \boxed{0}$$

3. [**15**] Given the function $f(x) = \ln(x+3)$.

   (a) [5 pts] State the domain and the range of f.

   $D(f) = \{x \,/\, x+3>0\} = \{x \,/\, x>-3\} = \underline{(-3,+\infty)}$

   $\text{Range}(f) = \underline{\mathbb{R}}$

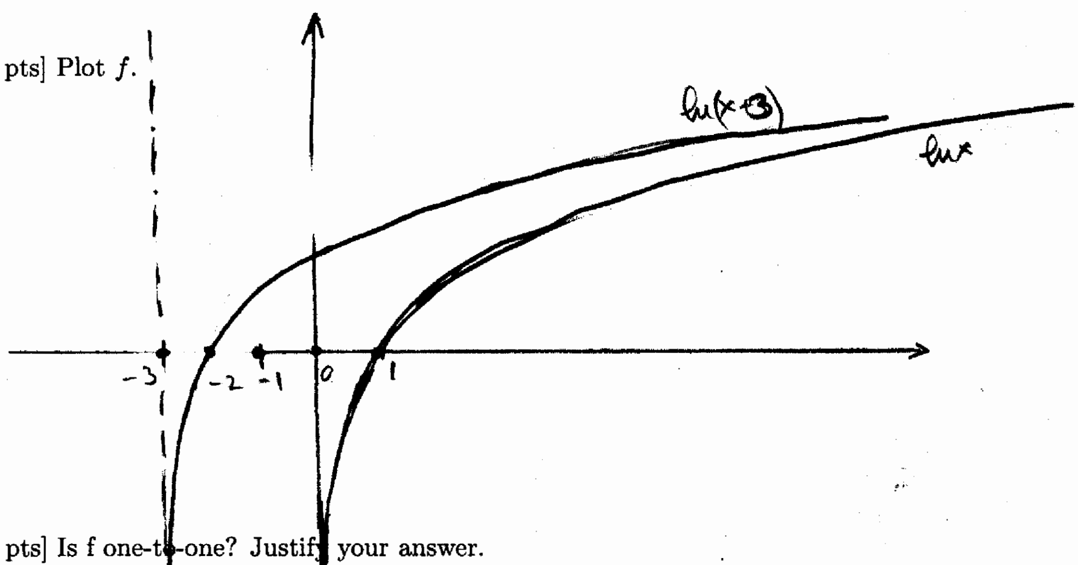
   (b) [5 pts] Find f^{-1}, i.e. the inverse function of f.

   $y = \ln(x+3)$ $\quad e^y = x+3$ $\quad x = e^y - 3$

   $f^{-1}(x) = e^x - 3$

   $\boxed{f^{-1}(x) = e^x - 3}$

   (c) [2 pts] Plot f.



   (d) [3 pts] Is f one-to-one? Justify your answer.

   Yes, f is one to one. ⇒ Any horizontal line crosses the graph of ln(x+3) in one point ⇒ According to HLT → f is one to one.

4. [**15**] Let us consider the parametric curve given by

$$x(\theta) = 4\cos(\theta) \qquad y(\theta) = 3\sin(\theta) \qquad \theta \in [0, 2\pi].$$

(a) [5 pts] Eliminate the parameter to find the Cartesian equation of the curve.

$$\left.\begin{aligned} \frac{x}{4} &= \cos\theta \\ \frac{y}{3} &= \sin\theta \end{aligned}\right\} \Rightarrow \left(\frac{x}{4}\right)^2 + \left(\frac{y}{3}\right)^2 = \cos^2\theta + \sin^2\theta = 1$$

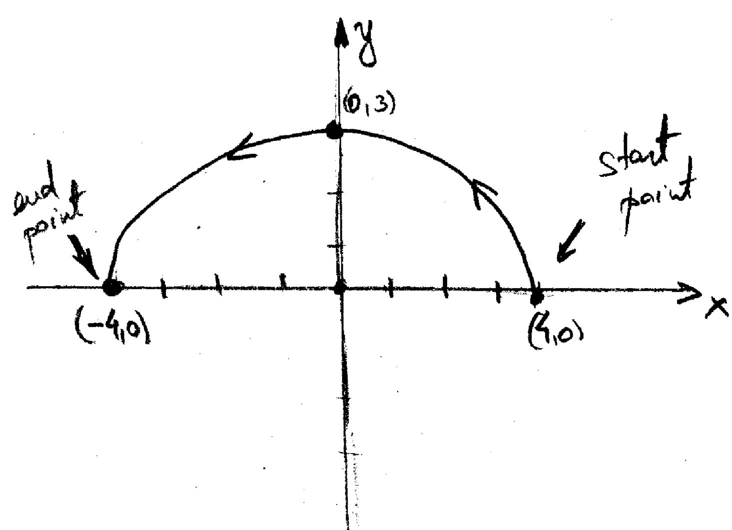
$$\boxed{\frac{x^2}{4^2} + \frac{y^2}{3^2} = 1}$$

Cartesian eq

(b) [5 pts] Identify the curve. ellipse

(c) [5 pts] Graph the curve for $\theta \in [0, \pi]$. Identify the starting and the ending points and draw an arrow on the curve to show the direction of increasing θ.

θ	x	y
0	4	0
$\frac{\pi}{2}$	0	3
π	-4	0



5. [**20**] Use the give graph to answer the follwoing questions

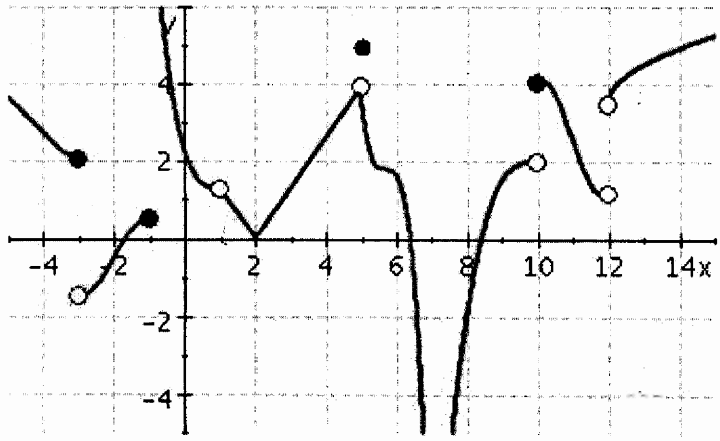


Figure 1: The graph of the function f

(a) [8 pts] State all x values where f is discontinuous.

$x = -3$ (Why? $\lim_{x\to -3} f(x)$ DNE since $\lim_{x\to -3^+} f(x) \neq \lim_{x\to -3^-} f(x)$)

$x = -1$ ($\lim_{x\to -1} f(x)$ DNE)

$x = 1$ (f is NOT defined at 1)

$x = 5$ ($\lim_{x\to 5} f(x) \neq f(5)$)

$x = 7$ ($\lim_{x\to 7} f(x)$ DNE)

$x = 10$ ($\lim_{x\to 10} f(x)$ DNE)

$x = 12$ (f is not defined)

(b) [7 pts] For each of the values stated in part a) determine whether f is continuous from the left, from the right or neither.

$x = -3$ f continuous from L

$x = -1$ f continuous from L

$x = 10$ f continuous from R

$x = 1$ neither

$x = 5$ neither

$x = 7$ neither

$x = 12$ neither

(c) [5 pts]List all values of x where the limit does not exists(∞, $-\infty$ or undefined).

$x = -3$ DNE

$x = -1$ DNE

$x = 7$ $\lim_{x\to 7} f(x) = -\infty$

$x = 10$ DNE

$x = 12$ DNE

6. [8] Find the equation of the tangent line to the graph of $y = \sqrt{x}$ at the point $(1, 1)$.

eq of tang line $\quad y - 1 = m(x-1)$.

$m =$ the slope

$$m = \lim_{x\to 1} \frac{\sqrt{x}-1}{x-1} = \lim_{x\to 1} \frac{\cancel{(x-1)}}{\cancel{(x-1)}(\sqrt{x}+1)} = \lim_{x\to 1} \frac{1}{\sqrt{x}+1} = \frac{1}{2}$$

$$\boxed{y - 1 = \frac{1}{2}(x-1)}$$

7. [7] Use the Squeeze theorem to compute

$$\lim_{x\to 0} \sqrt{x^4+x^2}\cos\left(\frac{\pi}{x}\right).$$

$$-1 \le \cos\frac{\pi}{x} \le 1 \qquad \Big| \sqrt{x^4+x^2}$$

$$-\sqrt{x^4+x^2} \le \sqrt{x^4+x^2}\cos\frac{\pi}{x} \le \sqrt{x^4+x^2}$$

$$\underbrace{\lim_{x\to 0} -\sqrt{x^4+x^2}}_{=0} \le \lim_{x\to 0} \sqrt{x^4+x^2}\cos\frac{\pi}{x} \le \underbrace{\lim_{x\to 0} \sqrt{x^4+x^2}}_{=0}$$

$\downarrow$

0

By the Squeeze Theorem $\Rightarrow$

$$\boxed{\lim_{x\to 0} \sqrt{x^4+x^2}\cos\frac{\pi}{x} = 0}$$