

Name: _____

Use of books, notes or calculators is **NOT** permitted. **Show all your work!** Answers without appropriate supporting work may not receive full credit. Clearly indicate your answers to each problem by underlining them or placing a **box** around your answers!

1. [10] Express the number $\frac{1}{2} + \frac{1}{\frac{-2}{3} + \frac{1}{4}}$ in simplest form (as an irreducible quotient of two integers).

$$\begin{aligned} \frac{1}{2} + \frac{1}{\frac{-2}{3} + \frac{1}{4}} &= \frac{1}{2} + \frac{1}{\frac{-8+3}{12}} = \frac{1}{2} + \frac{1}{\frac{-5}{12}} = \\ &= \frac{5}{2} - \frac{12}{5} = \frac{5-24}{10} = \boxed{-\frac{19}{10}} \end{aligned}$$

2. [10] Express as an integer the value of $\sqrt[3]{9}(3)^{-2/3}$.

$$\begin{aligned} \sqrt[3]{9} \cdot 3^{-2/3} &= 9^{1/3} \cdot 3^{-2/3} = (3^2)^{1/3} \cdot 3^{-2/3} = \\ &= 3^{2/3} \cdot 3^{-2/3} = 3^{2/3-2/3} = 3^0 = \boxed{1} \end{aligned}$$

3. [20] Solve the inequality $x^2 - 2x > 8$.

$$x^2 - 2x > 8 \quad \Leftrightarrow$$

$$x^2 - 2x - 8 > 0 \quad \Leftrightarrow (x-4)(x+2) > 0 \quad \Leftrightarrow$$

x	$-\infty$	-2	4	$+\infty$
$(x-4)$	---	---	0	+++++
$(x+2)$	-----	0	++++	++++
$(x-4)(x+2)$	++++	0	-----	0

$$x \in (-\infty, -2) \cup (4, +\infty)$$

4. [20] Solve the inequality $|2x + 3| \leq 5$.

Recall $|x| \leq c$ if and only if $-c \leq x \leq c$

$$|2x+3| \leq 5 \Leftrightarrow$$

$$-5 \leq 2x+3 \leq 5 \quad | -3$$

$$-8 \leq 2x \leq 2 \quad | :2$$

$$\boxed{-4 \leq x \leq 1} \quad \text{or}$$

$$\boxed{x \in [-4, 1]}$$

5. [20] For some real number θ , it is known that $\sin(\theta) = \frac{1}{2}$ and $\cos(\theta) > 0$. Compute $\sin 2\theta$.

$$\sin 2\theta = 2 \cdot \sin \theta \cdot \cos \theta = 2 \cdot \frac{1}{2} \cdot \cos \theta = ?$$

To compute $\cos \theta$ we use $\sin^2 \theta + \cos^2 \theta = 1$

$$\Rightarrow \cos^2 \theta = 1 - \sin^2 \theta = 1 - \left(\frac{1}{2}\right)^2 = \frac{3}{4} \Rightarrow$$

$$\cos \theta = \pm \frac{\sqrt{3}}{2} \quad \left. \begin{array}{l} \\ \cos \theta > 0 \end{array} \right\} \Rightarrow \cos \theta = \frac{\sqrt{3}}{2}$$

$$\text{THEN } \sin 2\theta = 2 \cdot \frac{1}{2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$$

6. [20] Plot the function $f(x) = -x^2 + 2x + 8$.

