**Name:** _____

Use of books, notes or calculators is **NOT** permitted.

**Please show all your work!** Answers without appropriate supporting work my not receive full credit.

Clearly indicate your answers to each problem by underlining them or placing a box around your answers!

Trigonometric functions at the values $0, \pi/6, \pi/4, \pi/3, \pi/2$, etc must be evaluated!

T/F Questions are graded with NO PARTIAL CREDIT.

Exam Score

| Problem | Score | Out of: |
|---|---|---|
| 1 | | 10 |
| 2 | | 20 |
| 3 | | 20 |
| 4 | | 10 |
| 5 | | 20 |
| 6 | | 20 |
| Total | | 100 |

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1. [ **10**] For the True/False questions below, clearly circle your answer.

   **T or F** If $f'(c) = 0$, then $f$ has a local maximum or minimum at $c$.

   **T or F** If $f'(x) < 0$ for $1 < x < 6$, then $f$ is decreasing on $(1, 6)$.

   **T or F** There exists a function $f$ such that $f(x) < 0$, $f'(x) < 0$, and $f''(x) > 0$ for all $x$.

   **T or F** $\lim_{x\to 1} \frac{\ln x}{x-1} = 1$.

   **T or F** If $f'(x)$ exists and is nonzero for all $x$, then $f(1) \neq f(0)$.

2. [ **20**] Two cars start moving from the same point. One travels south at 60 mi/h and the other travels west at 25 mi/h. At what rate is the distance between the cars increasing two hours later?

3. [**20**]Find the following limits. Verify if L'Hospital Rule applies, before using it.

   (a) [5 pts]

   $$\lim_{x\to 0} \frac{1-e^{2x}}{\cos x}$$

   (b) [5 pts]

   $$\lim_{x\to 0} \frac{e^x - 1 - x}{x^2}$$

   (c) [5 pts]

   $$\lim_{x\to \infty} x \sin\left(\frac{1}{x}\right)$$

   .

   (d) [5 pts]

   $$\lim_{x\to \infty} (x)^{\frac{1}{x}}$$

   .

4. [**10**] Find the linear approximation of the function $f(x) = \sqrt[3]{1+x}$ at $a = 0$, and use it to approximate $\sqrt[3]{1.06}$.

5. [ **20**] A rectangular container with an open top is to have a volume of 16 $m^3$. The length of the base is twice the width. Material for the base costs \$ 15 per square meter. Material for the sides costs \$ 10 per square meter.

   (a) [3 pts] Draw a picture of the open rectangular container and label on it the length, width and height in terms of the variables $x$ - the width of the base, $y$ - the height of the container.

   (b) [3 pts] Express the cost of materials for constructing the container as a function of $x$ and $y$.

   (c) [3 pts] Express the volume of the container as a function of $x$ and $y$.

(d) [3 pts] Express the cost of materials for the container as a function of $x$ *only*. Use the equations you found in (b) and (c) to do this.

(e) [8 pts] Find the dimensions of the container with least cost. Place your answers (width, length, height) in a box.

6. [ **20**] Let us consider the function $f(x) = x^3 - 6x^2 + 9x + 2$ on the closed interval $[-1, 4]$.

   (a) [10 pts] Find the absolute maximum and minimum values of $f$ and state where those values occur.

   (b) [10 pts] On what subinterval of $[-1, 4]$ is $f$ increasing/decresing? On what subinterval of $[-1, 4]$ is $f$ concave up/down? Sketch the graph of $f$.