Name: ____

Use of books, notes or calculators is **NOT** permitted. **Please show all your work!** Answers without appropriate supporting work my not receive full credit. Clearly indicate your answers to each problem by underlining them or placing a box around your answers! Trignometric functions at the values $0, \pi/6, \pi/4, \pi/3, \pi/2, \pi$, etc must be evaluated!

The exam consists of two parts. The first part contains 4 problems, all of which you must solve. Part 1 problems are worth 15 points each. The second part contains 4 problems. You must solve 3 of the 4. Part 2 problems are worth 15 points each. **Place an X in the box corresponding to the problem you DO NOT want graded. If you do not do so, we will randomly choose one for you.**

Exam Score

Problem	Score	Out of:
1		15
2		15
3		15
4		15
5		15
6		15
7		15
8		15
Total		105

PART 1: SOLVE ALL 4 OF THESE PROBLEMS

1. [**15p**] In each part find the indicated limit or explain why it doesn't exist. If the answer is $+\infty$ or $-\infty$, say so.

   (a) [**3p**] $\lim_{x\to\infty} \frac{2x^2-3x+1}{3x^2+5}$

   (b) [**3p**] $\lim_{x\to\infty} \frac{x^3-10x+1}{x^2-2}$

   (c) [**3p**] $\lim_{x\to 3^+} \frac{x^2-x+1}{x-3}$

   (d) [**3p**] $\lim_{x\to 1} \frac{1-\sqrt{x}}{1-x}$

   (e) [**3p**] $\lim_{x\to\infty} x^{\frac{1}{x}}$

2. [**15p**] Find each derivative:

   (a) [**4p**] $f'(x)$ where $f(x) = \sqrt[3]{x^4} + \ln x + \sin 2x$

   (b) [**4p**] $f'(x)$ where $f(x) = \sqrt{x} \arctan e^x$

   (c) [**4p**] $f'(1)$ where $f(x) = \frac{x^2+1}{e^x}$

   (d) [**3p**] $\frac{dy}{dx}$ where y is defined implicitly as a function of x through the equation

$$2 + x^2 = e^y + xy$$

3. [**15**] Evaluate each of the following integrals:

(a) [**4p**] $\int x^2 e^{x^3}\, dx$

(b) [**4p**] $\int \ln t^2\, dt$

(c) [**4p**] $\int_0^1 \frac{t}{t^4+1}\, dt$

(d) [**3p**] $\int_0^1 \frac{x^2}{(1+x^3)^2}\, dx$

4. [**15**] Consider the function $f(x) = x^3 + 3x^2 - 9x$.

   (a) [**4p**] Compute $f'(x)$ and find all critical points of $f(x)$. Determine the intervals on which $f(x)$ increasing/decresing?

   (b) [**4p**] Compute $f''(x)$ and determine the intervals on which $f(x)$ is concave up/concave down. Also identify any inflexion points.

   (c) [**4p**] At what x values is there a relative maximum of f? At what x values is there a relative minimum of f?

   (d) [**3p**] Sketch the graph of f capturing the key information you found in (a) - (d).

PART 2: SOLVE 3 of OF THESE 4 PROBLEMS

5. [**15p**] The management of a large store wishes to add a fenced-in rectangular storage yard of 20,000 square feet, using the building as one side of the yard. Find the minimum amount of fencing that must be used to enclose the remaining 3 sides of the yard.

   (a) [2 pts] Draw a picture of the rectangular storage yard and the building. Label with x and y the sides of interest.

   (b) [2 pts] Express the amount of fencing as a function of x and y.

   (c) [2 pts] Express the area of rectangular storage yard as a function of x and y.

   (d) [3 pts] Express the amount of fencing as a function of x *only*. Use the equations you found in (b) and (c) to do this.

   (e) [6 pts] Find the dimensions of the rectangular storage yard with least fencing. Be sure to justify your answer with an derivative test. Place your answers (length, width) in a box.

Continue problem 5 here

6. [**15p**] A 10 feet long ladder is leaning against the side of a house. The foot of the ladder is pulled away from the house at a rate of 4feet/sec. Determine how fast the top of the ladder is moving when the foot of the ladder is 6 feet away from the house. Be sure to include in the space below a sketch and a definition of all variables used.

7. [**15p**] Find the area of the region R bounded by the graph of the function $y = x^2\sqrt{x-1}$, the lines $x = 1$ and $x = 2$ and the x-axis.

8. [**15p**] Use a linear approximation to estimate $\sqrt{16.04}$. Be sure to write down the linear approximation formula, the function chosen and the point where you linearize.