Mean Value Thm

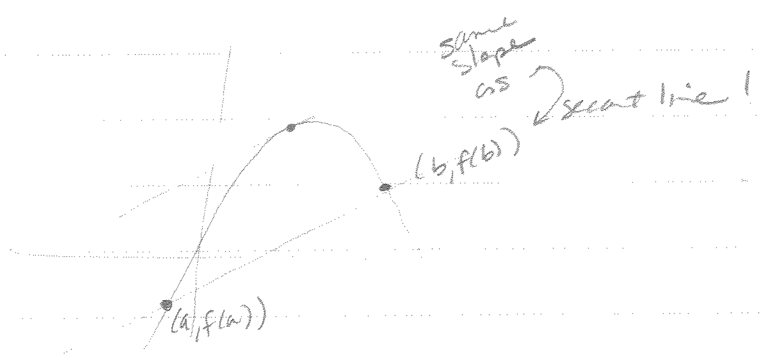
If f is cont. on $[a,b]$ and diff on (a,b), then there exists a number c in (a,b) such that

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

mean rate of change

alternate form:

$$f(b) = f(a) + (b-a)f'(c$$



Does MVT apply? If so find all c values in (a,b)

note polynom, rational, transcendental cont & diff for all of domain

$f(x) = \frac{x+1}{x}$ on $[1/2, 2]$

Domain $= \{x \mid x \neq 0\}$ ok

$f(x) = 1 + 1/x \qquad f'(x) = -1/x^2$

$$f'(c) = \frac{f(2) - f(1/2)}{2 - 1/2} = \frac{1 + \frac{1}{2} - (1+2)}{3/2} = \frac{-3/2}{3/2} = -1$$

$-1 = -1/x^2 \qquad x^2 = 1 \qquad x = \pm 1$ but only $\underline{\underline{x=1}}$ in interval

The height of a ball t seconds after it is thrown upward from height 32 ~~feet~~ and w/ an initial velocity 48 ft/s is

$$f(t) = -16t^2 + 48t + 32$$

a) find average velocity of object during 1st 2 seconds

$$\text{avg velocity} = \frac{f(2) - f(0)}{2-0} = \frac{-16(4) + 48(2) + \cancel{32} - \cancel{32}}{2} = \frac{-64 + 96}{2} = \frac{32}{2} = 16 \text{ ft}$$

b) Use MVT to verify that at some time during 1st 2 sec the inst velocity = avg vel above. Find that time

to apply MVT need f cont ~~diff~~ on [0,2] ✓
f diff on (0,2) ✓

polynomial so cont & diff everywhere!

so MVT applies
∃ some c in (0,2) such that

$$f'(c) = \frac{f(2) - f(0)}{2-0}$$

$$f' = -32t + 48$$

$$-32c + 48 = 16$$

$$-32c = -32$$

$$\boxed{c = 1 \text{ sec}}$$

M1010

4.3, 4.5 Shapes of graphs:

<u>1st deriv. tells us something about shape</u>

<u>Thm/Test</u>

If $f'(x) > 0$ (slope of tan pos) on interval then f is increasing on <u>inter.</u>

If $f'(x) < 0$ (slope of tan neg) then f is decreasing

1st Deriv Test for local extrema

Suppose c a critical # of cont. function f

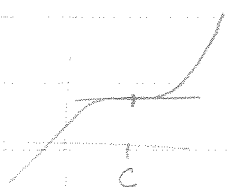
1) If $f' > 0$ to left of c ; $f' < 0$ to right of c

$+ \quad c \quad -$ sign of f' then f has local max at c

2) If $f' < 0$ to left of c ; $f' > 0$ to right of c

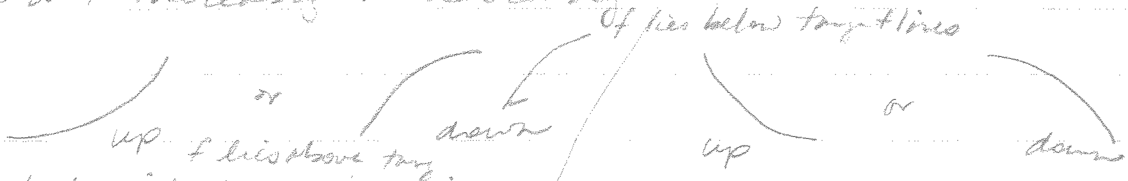
$- \quad c \quad +$ sign of f' then f has local min at c

3) If f' same sign on both sides of c, then no local max or local min at c!



zero slope → but no extrema!

How is f increasing or decreasing



has to do with concavity

<u>Concavity test</u>

If $f'' > 0$ on interval then f concave up on I
(f' increasing)

If $f'' < 0$ on interval then f concave down on I
(f' decreasing)

A point where concavity changes from up to down or vice versa called <u>inflection point</u>

(cand. are $f''(c)=0$ or $f''(c)$ undefined)

2^{nd} Deriv. Test for local Extrema

Suppose f'' continuous near c

1) If $f'(c)=0$ (crit #) and $f''(c)>0$ (concave up) then f has local min at c

2) If $f'(c)=0$ (crit #) and $f''(c)<0$ (concave down then f has local max at c

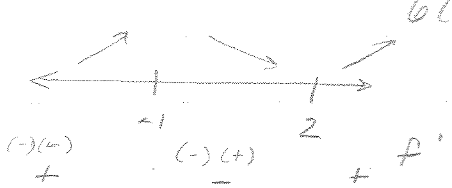
Determine $\uparrow\downarrow$ $\cup$ $\cap$ from graph given $\rightarrow$

or start w/ a function and find some or all of <u>following</u>

$f(x)=2x^3-3x^2-12x$ a) int. of inc. or decrease

$f' = 6x^2-6x-12 = 6(x^2-x-2)$

$6(x-2)(x+1)$ crit #s $x=2$, $x=-1$



inc. on $(-\infty,-1)$ and $(2,\infty)$

decr. on $(-1,2)$

b) local max & min values

local max at -1 (1^{st} der. test) of $f(-1) = -2-3+12 = 7$

local min at 2 (1^{st} der. test) of $f(2) = 16-3\cdot 4-24 = -20$

c) intervals of concavity & inflection pts

$f''(x) = 12x-6 = 0 \Rightarrow 12x=6$

$x = 6/12 = 1/2$

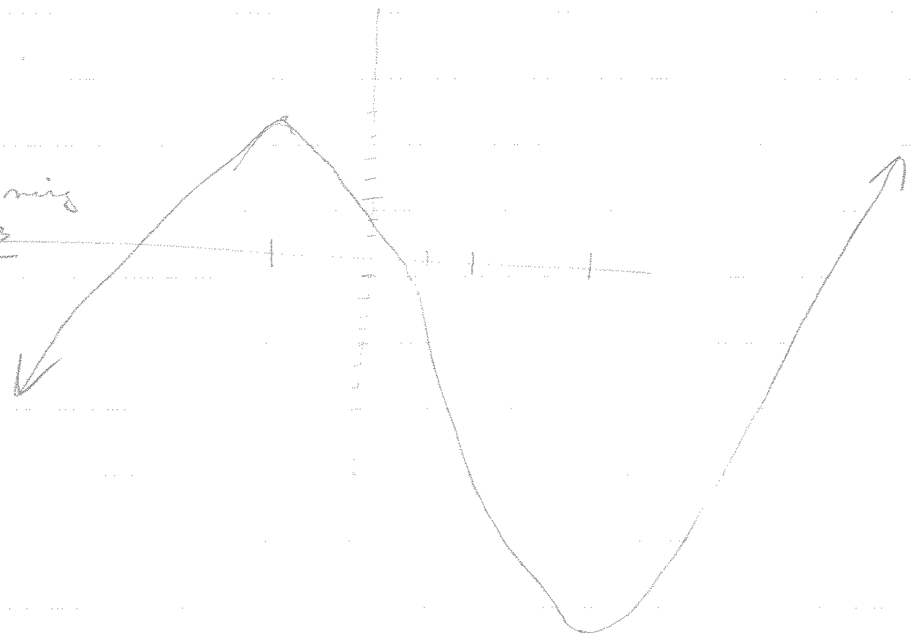
$-\quad \frac{1}{2} \quad + \qquad f''$

concave down on $(-\infty, \frac{1}{2})$ concave up on $(\frac{1}{2}, \infty)$
$f'' < 0$

$x = \frac{1}{2}$ inflection pt.

Sketch graph

Improve by determining intercepts



$f(x) = \cos^2(x) - 2\sin(x)$ on $0 \le x \le 2\pi$

a) int of ↑ ↓
b) local max, min
c) concavity & inflect

$f' = 2\cos(x)(-\sin(x)) - 2\cos(x)$

$= -2\cos(x)\,(\underbrace{\sin(x)}_{\geq 0} + 1) = 0$ $\qquad \sin(x) = -1$

crit # $\qquad \cos(x) = 0 \Rightarrow \frac{\pi}{2}, \frac{3\pi}{2}$ $\qquad x = \frac{3\pi}{2}$

$0 \quad - \quad \frac{\pi}{2} \quad + \quad \frac{3\pi}{2} \quad - \quad 2\pi$

local min of -2 $\qquad$ local max of 2

so f inc. on $(\frac{\pi}{2}, \frac{3\pi}{2})$

dec on $[0, \frac{\pi}{2})$ &

$(\frac{3\pi}{2}, 2\pi)$

$\cos^2(\frac{\pi}{2}) - 2\sin(\frac{\pi}{2})$

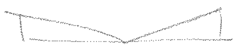
c) $f'' = 2\sin(x)(\sin(x)+1) - 2\cos(x)(\underbrace{\cos(x)}_{})$ $\qquad 2(2\sin(x) - 1)(\sin x + 1)$

$\frac{\cos^2(x)}{1-\sin^2(x)}$ $\qquad 4\sin^2(x) + 2\sin(x) - 2$

$= 2\sin^2(x) + 2\sin(x) - 2(1 - \sin^2(x))$ $\qquad 2(2\sin^2(x) + \sin(x) - 1)$

possible inf: $2\sin(x) - 1 = 0$ $\quad\quad$ $\sin x = -1$

$\sin(x) = 1/2$ $\quad\quad$ $x = 3\pi/2$



$\pi/6$; $5\pi/6$

$0 \quad \pi/6 \quad 5\pi/6 \quad 3\pi/2 \quad 2\pi$

	$(0,\pi/6)$	$(\pi/6,5\pi/6)$	$(5\pi/6,3\pi/2)$	$(3\pi/2,2\pi)$	f''
	−	+	−	−	$2\sin(x) - 1$
	+	+	+ 0	+	$-\sin(x) + 1$
	−	+	−	−	

Concave down on $(0, \pi/6)$ $(5\pi/6, 3\pi/2)$ and $(3\pi/2, 2\pi)$

" up on $(\pi/6, 5\pi/6)$

2 inflect. pts

→ $(\pi/6, -1/4)$ $(5\pi/6, -1/4)$

$\left(\frac{\sqrt{3}}{2}\right)^2 - 2\left(\frac{1}{2}\right)$ $\quad \frac{3}{4} - 1$ $\quad\quad$ $\left(\frac{\sqrt{3}}{2}\right)^2 - 2\left(\frac{1}{2}\right)$

$\frac{3}{4} - 1$

4.5 Curve sketching

Adds to ↑↓ $\cup$ $\cap$ inflection pts

- Domain of f
- x & y intercepts $\quad$ y int sub in $x = 0$

  x-intercepts, solve eqn $= 0$
- any info on symmetry we may know or periodicity
- asymptotes horizontal & vertical

  $\lim_{x \to \pm\infty}$ $\quad\quad$ denom undefined

Sketch $y = \frac{x}{x^2-9}$

$D: \{x \mid x \neq 3, x \neq -3\}$

$y\text{-int} \to 0$

$0 = \frac{x}{x^2-9} \Rightarrow x = 0$ x int

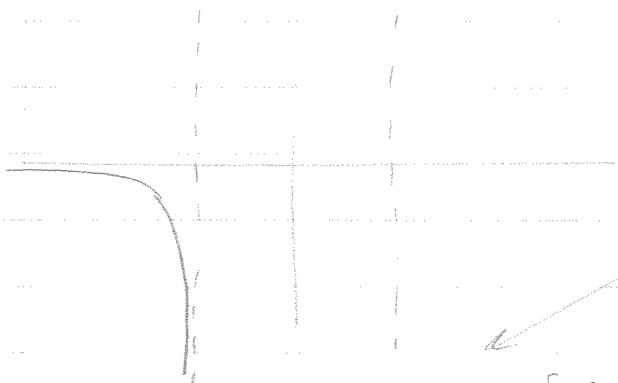
$\lim_{x\to\infty} \frac{x}{x^2-9} = 0 \quad \lim_{x\to-\infty} \frac{x}{x^2-9} = 0 \quad y=0$ HA

$x=3 \quad x=-3$ VA $\quad \lim_{x\to 3^-} -\infty \quad \lim_{x\to 3^+} +\infty$

$\lim_{x\to -3^-} = +\infty \quad \lim_{x\to -3^+} -\infty$

crit #'s

$f' = \frac{(x^2-9) - x(2x)}{(x^2-9)^2} \quad -\frac{x^2+9}{(x^2-9)^2} < 0$ all x so decreasing everywhere in domain



f'' concavity

$(x^2-9)^2 - (x^2+9)\,2(x^2-9)$

$-\frac{2x(x^2-9)^2 - (x^2+9)\,2(x^2-9)(2x)}{(x^2-9)^4}$ simplifies

$= \frac{2x(x^2+27)}{(x^2-9)^3} > 0$

$\frac{2x\cancel{(x^2-9)}\,[x^2-9-2(x^2+9)]}{(x^2-9)^{\cancel{4}3}}$

x^2-9-2x^2-18

$(-x^2-27)$

$= \frac{2x(x^2+27)}{(x^2-9)^3}$

	-3		0		3	
−		−		+		+
+		+		+		+
+		−		−		+
−		+		−		+