Experimental Measurements of the Dynamic Electric Field Topology Associated with Magnetized RF Sheaths

Elijah H. Martin$^1,2$
Steve Shannon$^1$ – John Caughman$^2$
Ralph Isler$^2$ – Chris Klepper$^2$

$^1$Department of Nuclear Engineering, NCSU, Raleigh NC  -----  $^2$Fusion Energy Division, ORNL, Oak Ridge TN
Outline

- **Introduction**
  - RF Sheaths – Ion Cyclotron Range of Frequency Antenna
    - Collisionless Heating
  - Optical Emission Spectroscopy and the *dynamic* Stark effect

- **DStarVE – Dynamic Stark Verification Experiment**
  - Experimental Setup
  - Collection Optics and Spectroscopic System

- **Initial DStarVE Results**
  - Time Averaged Optical Emission Spectroscopy – H$_2$
  - Phase Resolved Optical Emission Spectroscopy – H$_2$

- **Conclusions and Future Work**
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RF Sheaths – ICRF Antenna and Collisionless Heating

- The underlying motivation for this research is to quantify the effect of the magnetized RF sheath on the operation of ICRF antenna system implemented on a thermonuclear reactor.

- ICRF antenna systems are a critical component!

- RF sheaths are formed on the faraday screen/antenna box.

- Power is absorbed in the RF sheath/presheath through collisional and collisionless processes – undesirable effects on ICRF antenna operation.

- Measurement of electric field topology and dynamic will aid the development of the theory utilized to model ICRF antenna near field – plasma interactions.

- What is the effect of the electric field topology and dynamic on the heating/power absorption mechanism associated with the RF sheath.

  Kinetic Effect – Fermi Acceleration
  – Two Stream Instability

  Fluid Effect – Compression/Rarefaction of Sheath
  – Ohmic

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OES and the Dynamic Stark Effect

- The electric field associated with the RF sheath is experimentally measured by utilizing time averaged and phase resolved optical emission spectroscopy.

- The local electric and magnetic field parameters are determined from the measured line profile of a properly chosen electronic transition.

**Balmer series transitions**

- The electric field associated with an RF sheath can be written in terms of a Fourier series:

\[ \overline{E}(t) = \overline{E}_o + \sum_{n=1}^{\infty} \overline{E}_n \cos(n \omega t + \phi_n) \]

- The line profile can be calculated utilizing one of two methods:

  **Floquet Method**  \[ \Delta \approx \frac{A_{ki}}{\omega} \ll 1 \]

  **Quasi-static Method**  \[ \Delta \approx \frac{A_{ki}}{\omega} \gg 1 \]

- The results to follow were obtained assuming the quasi-static method is valid.

- The dynamic Stark effect is a multi-photon process. Photons associated with the atomic transitions are emitted/absorbed with photons associated with the oscillating electric field!
Dynamic Stark Effect – Quasi-static Method

- In the limit $\Delta \gg 1$ the quantum states respond instantaneously to the electric field and the quasi-static method can be utilized to line profile.

$$i\hbar \frac{\partial \Phi}{\partial t} = H^* \Phi \quad H^* = H^0 + \frac{\mu_B}{\hbar} \mathbf{B} \cdot [\mathbf{J} + \mathbf{S}] + e\mathbf{E}_i \cdot \mathbf{r}$$

$$\bar{E}_i = \bar{E}_0 + \sum_{n=1}^{NH} E_n \cos(n \omega t_i + \phi_n) \quad t_i \in [0, \tau]$$
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DStarVE Experimental Setup

RF Biased Electrode
- 13.56 MHz -

RF Compensated Langmuir Probe
2.45 GHz Directional Couplers
RF Directional Couplers

Capacitive and Inductive Probe
DC Bias Probe
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Collection Optics and Spectroscopic System

Subtraction of a ‘unperturbed’ line profile can lead to significant errors when extracting electric field parameters.

No Unperturbed Emission Collected

Optics aligned with dual slit assembly

- Mechanical Slit
- GT Polarizer
- Mechanical Slit

Spectrometer
- 0.5 m Czerny - Turner
- triple turret grating

Camera
- PI-MAXIII Intensified CCD
- 3.68 ns gate width
- 1 MHz repetition rate

0.03 nm FWHM Gaussian instrument function
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- **Conclusions and Future Work**
In order to accurately extract the electric field parameters from the line profile a robust fitting algorithm is required.

Due to the large number of free parameters the computational time and accuracy of the fit will be increased by utilizing multiple constraints:

- Line profiles of multiple transitions: H$\alpha$, H$\beta$
- Known polarizations: $\sigma$ and $\pi$
- Subtraction of impurity transitions

A simple fitting routine for the H$\alpha$ line profile is currently utilized while the robust algorithm is being developed.

\[ \vec{E}(t) = E_o \hat{z} + E_o \cos(\omega t) \hat{\omega} \]

\[ \vec{B} = B_o \hat{z} = 0.15 \ T \]
**Time Averaged OES – Electric Field Calculation**

- Utilizing the FWHM of the Gaussian fit to the simulated $\sigma$ and $\pi$ profiles the following relationships are found.

\[
E_o = a\left[\sigma_{FWHM}^2 - \pi_{FWHM}^2\right]^{1/2} + b
\]

\[
\pi_{FWHM} = aE_o^3 + bE_o^2 + cE_o + d
\]

\[
\sigma_{FWHM} = aE_o + b
\]

- The above method will allow for the neutral temperature to be determined.

- $\sigma$ and $\pi$ profiles should give $kT_n \sim 1$ eV if excitation path is through Franck-Condon dissociation.

**Method Requirements**

- $\sigma$ and $\pi$ profiles must have same $kT_n$

- Gaussian line profile
Time Averaged OES – RF Sheath Parameters

- The electric field is calculated from the $\sigma_{\text{FWHM}}$ and $\pi_{\text{FWHM}}$ associated with the ‘thermal’ group of atoms – **method requirements are met**.

- The sheath voltage obtained by integrating the experimentally determined electric field is in good agreement with the electrical probe measurements.

**OES**

- Electrode DC Voltage: $-2.38 + V_{\text{PDC}}$ kV
- Electrode RF Voltage: $2.38 + V_{\text{PRF}}$ kV
- Sheath Thickness: 1.13 cm

**Electrical Probe**

- Sheath Voltage: $-2.55$ kV
- Sheath Thickness: 1.13 cm
Time Averaged OES – H\textsubscript{\beta} Line Profile

- The H\textsubscript{\beta} transition is more sensitive to the electric field – analysis will yield accurate electric field time dependence – will require robust fitting algorithm.

- Two temperature distribution is not observed in the H\textsubscript{\beta}.
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Phase Resolved OES

- Phase resolved measurements of total emission were conducted to gain insight on the RF sheath parameters presented above.

- Phase resolved images are very similar to those obtained by T. Gans et al.[4] and D. O’Connell et al.[5].

60 temporal points – 0 to 73.8 ns
5 ns resolution
16 spatial points – 0.9 to 8.5 mm
0.75 mm resolution

- Total emission is dominated by the H$_\alpha$ transition for our system bandwidth of 350 to 800 nm.

Phase Resolved OES – Impact on Electric Field Calculations

- The time dependence of the emission intensity **must** be taken into account in the calculation of the line profile.

- Constant emission intensity assumption will underestimate the electric field parameters.

- An electron-electron two-stream instability\(^\text{[6]}\) seems to be present in the PROES data and the discharge current – driven at 163 MHz.

The electron beam generated due to the sheath expansion correlates with the calculated electric field.

The dramatic change in the line profile of the H$_\beta$ transition measured at 1 and 2 mm may indicate field reversal.

Emission from the ‘fast’ group of neutrals has a 40 per cent higher peaking factor than the ‘thermal’ group.
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Conclusions and Future Work

- An initial estimate of the dynamic electric field topology associated with a magnetized capacitively coupled RF sheath has been successfully measured based on the H\textsubscript{\alpha} line profile.
  - The electric field strength is approximately a linear function of space for the given model.
  
  \[
  \vec{E}(t) = E_o \hat{z} + E_o \cos(\omega t) \hat{z} \quad \rightarrow \quad E_o \in [3.2, 1.1] kV/cm \quad z \in [1, 8] mm
  \]

- Phase resolved optical emission spectroscopy reveals a strong time dependence in the total emission intensity in the sheath.
  - In order to accurately determine the electric field parameters in the RF sheath this must be taken into account in the line profile calculations: **Floquet Method – Quasi-static Method**.

- Determine if the quasi-static method approximates the atomic physics well in the RF range of frequencies for the Balmer series transitions\textsuperscript{[7]}.

- Develop robust line fitting algorithm based on the following constraints:
  - Two line profiles: H\textsubscript{\alpha} H\textsubscript{\beta}
  - Two polarizations: \(\sigma\ \pi\)
  - Time dependence of transition intensity

**EXPERIMENTS!!!**

\textsuperscript{[7]} Calculation of RF Field Characteristics using Non-perturbative Optical Diagnostics with a Generalized Dynamic Stark Effect Model – **IP3N-48 Wendesday 13:00-15:00 CC12 A-D**
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QUESTIONS
Dynamic Stark Effect – Floquet Method

In the limit $\Delta \ll 1$ the calculation of the line profile requires a solution of the following Schrödinger equation:

$$i\hbar \frac{\partial \Phi}{\partial t} = H^* \Phi + \sum_{n=1}^{NH} H_{n}^{RF} \left[ e^{i(n\omega t + \phi_n)} + e^{-i(n\omega t + \phi_n)} \right] \Phi$$

$$H^* = H^o + \frac{\mu_B}{\hbar} B \cdot [\vec{J} + \vec{S}] + e\overline{E}_o \cdot \overline{r} \quad H_{n}^{RF} = \frac{e\overline{E}_n \cdot \overline{r}}{2}$$

$$\overline{E}(t) = \overline{E}_o + \sum_{n=1}^{NH} \overline{E}_n \cos(n\omega t + \phi_n)$$

Utilizing the Floquet theorem\textsuperscript{[2]} and a Fourier series expansion the time dependent Schrödinger equation can be reduced to an eigenvalue problem – the quantum states are stationary\textsuperscript{[3]}.

Computational intensive for the RF range of frequencies.

Convergence to quasi-static method in RF range of frequencies due to finite resolution of spectroscopic system??? --- Currently under investigation.

\textsuperscript{[2]} C. M. Bender and S. A. Orszag, \textit{Advanced Mathematical Methods for Scientists and Engineers – Asymptotic Methods and Perturbation Theory}, Springer-Verlag NY Inc. 1999

\textsuperscript{[3]} W. W. Hicks, R. A. Hess, and W. S. Cooper, Physical Rev. A 5 (1972) 490
Dynamic Stark Effect – Quasi-static Method

- In the limit $\Delta \gg 1$ the quantum states respond instantaneously to the electric field and the quasi-static method can be utilized to line profile.

$$i\hbar \frac{\partial \Phi}{\partial t} = H^* \Phi \quad H^* = H^o + \frac{\mu_B}{\hbar} \overline{B} \cdot [\overline{J} + \overline{S}] + e \overline{E}_i \cdot \overline{r}$$

- In the RF range of frequencies the satellite structure saturates the line profile – due to finite resolution of spectroscopic system – for the following conditions:

$$\overline{E}_i = \overline{E}_o + \sum_{n=1}^{NH} \overline{E}_n \cos(n\omega t_i + \phi_n) \quad t_i \in [0, \tau]$$

- The results to follow were obtained assuming the quasi-static method is valid for the Balmer series transitions in the RF range of frequencies.

$$FWHM \equiv 0.025 \text{ nm}$$

$\sim 5 \text{ GHz}$
The Dynamic Stark Effect – Degenerate States

Consider a one electron atom in the presence of a perturbative monochromatic electric field, the time dependent Schrödinger equation takes the form:

\[ i\hbar \frac{\partial \Psi}{\partial t} = H^0 \Psi + e\mathbf{r} \cdot \mathbf{E}_{RF} \cos(\omega t) \Psi \]

Ignoring the ‘fine-structure’ operators in the unperturbed Hamiltonian Blochinzew\(^7\) arrived at the first order correct result utilizing time dependent degenerate perturbation theory.

\[
\left\langle I_{nn}(\nu^*) \right\rangle = \sum_{\mathbf{m}} \left| \langle \phi_{n n_n n_m m} | \mathbf{E} \cdot \mathbf{r} | \phi_{n'n_n'n_m'm} \rangle \right|^2 \sum_{p=-\infty}^{\infty} J^2_p(\alpha X) \delta(\nu^* - \nu_{nn} - p\nu) \quad \text{Time Averaged Line Profile}
\]

\[
X = n(n_1 - n_2) - n'(n'_1 - n'_2)
\]

\[
\alpha = \frac{3\hbar E_{RF}}{2m_e e c \omega}
\]

\[
\nu = \frac{\omega}{2\pi}
\]

The Dynamic Stark Effect – Non-Degenerate States

- Consider a one electron atom in the presence of a static electric field, static magnetic field, and monochromatic electric field. The time dependent Schrödinger equation takes the form:

\[
i\hbar \frac{\partial \Phi}{\partial t} = H^* \Phi + H^{RF} \left[ e^{i\omega t} + e^{-i\omega t} \right] \Phi
\]

- Floquets theorem\[8\] asserts that a differential equation with periodic coefficients will have a solution of the form:

\[
\Phi(t) = T(t)e^{-i\lambda t}
\]

- We expand the periodic coefficients, \( \tau(t) \), in a Fourier series:

\[
\Phi_k(t) = \sum_{s=-\infty}^{\infty} \sum_{j=1}^{N} c_{kj}^s \Psi_j e^{-i(\lambda_k + s\omega)t}
\]

- We choose the coupled basis set to work within and expand \( T(t) \):

\[
\Phi_k(t) = \sum_{j=1}^{N} \tau(t)_{kj} \Psi_j e^{-i\lambda_k t}
\]

- Redefine \( \lambda_k \):

\[
\Phi_k(t) = \sum_{s=-\infty}^{\infty} \sum_{j=1}^{N} c_{kj}^s \Psi_j e^{-i\left(\frac{E_k + s\omega}{\hbar}\right)t}
\]

The Dynamic Stark Effect – Non-Degenerate States

Substituting this result into the Schrödinger equation we perform the following integral using the orthonormality of our basis set:

\[ \langle \Psi_l \mid i\hbar \frac{\partial \Phi_k}{\partial t} \rangle = \langle \Psi_l \mid H^* \Phi_k \rangle + \langle \Psi_l \mid H^{RF} e^{i\omega t} \Phi_k \rangle + \langle \Psi_l \mid H^{RF} e^{-i\omega t} \Phi_k \rangle \]

The resulting equation must be satisfied at all time thus we can equate equal powers of:

\[ e^{-i\omega t} \]

We arrive at an infinite set of algebraic equations:

\[
\left( E_k + s\hbar \omega \right) c_{kl}^s = \sum_{j=1}^{N} c_{kj}^{s-1} W_{lj}^{RF} + \sum_{j=1}^{N} c_{kj}^{s} W_{lj}^{*} + \sum_{j=1}^{N} c_{kj}^{s+1} W_{lj}^{RF} \]

\[ k = 1..N \]
\[ l = 1..N \]
\[ s = \infty..-\infty \]

In order to solve this system of algebraic equations we truncate the set such that:

\[ c_{kj}^s = 0 \]

\[ |s| > s_m \]

The truncated set of algebraic equations can be written in matrix form:
The Dynamic Stark Effect – Non-Degenerate States

- Substituting this result into the Schrödinger equation we perform the following integral using the orthonormality of our basis set:

\[
\begin{bmatrix}
W^* + \text{i} S \ h \omega \\
W^{RF} \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
W^* + \text{i} (s - 1) \ h \omega \\
W^{RF} \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
W^{RF} \\
W^* - \text{i} S \ h \omega
\end{bmatrix}
\begin{bmatrix}
\vec{c}^{-s_m} \\
\vec{c}^{-s_m+1} \\
\vec{c}^{-s_m} \\
\vec{c}^{-s_m+1}
\end{bmatrix}
- \vec{E}_k
= 0
\]

\[
W = \begin{bmatrix}
W_{11}^* & \cdots & W_{1N}^* \\
\vdots & \ddots & \vdots \\
W_{N1}^* & \cdots & W_{NN}^*
\end{bmatrix}
\]

\[
W^{RF} = \begin{bmatrix}
W_{11}^{RF} & \cdots & W_{1N}^{RF} \\
\vdots & \ddots & \vdots \\
W_{N1}^{RF} & \cdots & W_{NN}^{RF}
\end{bmatrix}
\]

- The \textbf{W-matrices are} \textbf{N} x \textbf{N} – \textbf{N} is the number of basis states with a given \textbf{principal quantum number}, \textbf{n}.

- The complete solution to the Schrödinger equation for a given \textbf{n} must have \textbf{N} basis states.
The Dynamic Stark Effect – Time Averaged Line Profile

The time averaged line profile for spontaneous emission is given by:

\[
\langle I_{kk}(v^*) \rangle \propto \sum_{s_1=-s_m}^{s_m} \sum_{s_2=-s_m}^{s_m} \delta\left(v^* - \frac{(E_k - E_{l1})}{h} - (s_2 - s_1)\nu\right) \sum_{s_1=-s_m}^{s_m} \sum_{s_2=-s_m}^{s_m} \delta(s_2 - s_1 - s_2 + s_1) \left( \sum_{i=1}^{N_{nf}} c_{li}^s \Psi_i \right) \left( \sum_{j=1}^{N_{ni}} c_{kj}^{s_j} \Psi_j \right)^* \left( \mathbf{E} \cdot \mathbf{F} \right) \left( \sum_{j=1}^{N_{ni}} c_{kj}^{s_j} \Psi_j \right) \]

- \( n_f \) – principal quantum number of final state
- \( n_i \) – principal quantum number of initial state
- \( N_{nf} \) – number of basis states with principal quantum number \( m \)
- \( N_{ni} \) – number of basis states with principal quantum number \( n \)

To calculate the line profile for an RF electric field in the range of 50 MHz might require 500 to 1000 Floquet blocks \((2s_m+1)!\) For the \( H_\beta \) transition:

- \( n_f = 2 \quad N_{nf} = 8 \quad \rightarrow \quad \text{Floquet matrix is 8000 x 8000} \)
- \( n_i = 4 \quad N_{ni} = 32 \quad \rightarrow \quad \text{Floquet matrix is 32000 x 32000} \)
Dynamic Stark Effect – Code Validation

- In order to validate the code the results of the Floquet and Blochinzew analyses must be equivalent when considering degenerate states.

\[ \overline{E(t)} = E_o \hat{z} + E_1 \cos(\omega t) \hat{z} + E_2 \cos(2\omega t + \phi_2) \hat{z} \]

\[
E_o = 2 \text{ kV/cm} \quad \omega / 2\pi = 20 \text{ GHz} \\
E_1 = 4 \text{ kV/cm} \quad \phi_2 = \pi / 6 \\
E_2 = 2 \text{ kV/cm}
\]

- The Floquet and Blochinzew analyses agree for all valid field configurations!
Optically Resolving the RF Sheath – Alignment

- In order to achieve the spatial resolution set by the collection optics the field of view must be precisely aligned parallel to the electrode surface.

- A 2D ray trace program was utilized to determine machining tolerances and simulate the alignment process.