MA (ST) 412 Actuarial Mathematics, 7.1, 7.2, 7.4, and 7.31 Solutions

Note: Supplemental problem was worked during the lecture on 7.4.

1. (7.1) From Example 7.1.1, we have

$$_1V = 0.15111.$$

In addition, we can get

$$Pr(K = k|K \geq 2) = \frac{Pr(K = k)}{Pr(K \geq 2)} = \frac{0.2}{0.2 + 0.2 + 0.2} = \frac{1}{3}, \quad k = 2, 3, 4.$$

Therefore, we have

$$_2V = E[\,_2L|K \geq 2] = \frac{1}{3}(0.64067 + 0.30169 - 0.01811) = 0.30809,$$

where the values of $_2L$ for $k = 2, 3, 4$ are from the first table on page 205. Similarly, we can get

$$_3V = \frac{1}{2}(0.64067 + 0.30169) = 0.47118, \quad _4V = 0.64067.$$

2. (7.2) From Example 7.1.1, we have

$$_1V = 0.14925.$$

Similar to the solution for (7.1), we have

$$Pr(K = k|K \geq 2) = \frac{1}{3}, \quad k = 2, 3, 4.$$

Therefore, we can get

$$E[e^{0.1\,_2L}|K \geq 2] = \frac{1}{3}(1.06579 + 1.02992 + 0.99718) = 1.030963,$$

where the value of $e^{0.1\,_2L}$ for $k = 2, 3, 4$ are from the second table on page 205. Thus, we have

$$_2V = 10 \log E[e^{0.1\,_2L}|K \geq 2] = 10 \log 1.030963 = 0.30493.$$

Similarly, we can get

$$_3V = 0.46745, \quad _4V = 0.63716.$$

3. (7.4) By assumption, for wealth $w = 10$, ${}_1V$ should satisfy

$$u(w - {}_1V) = E[w - {}_1L].$$

That is

$$w - {}_1V - 0.01(w - {}_1V)^2 = E[w - {}_1L - 0.01(w - {}_1L)^2].$$

Plug in $w = 10$, and we can get

$$ {}_1V^2 - 80\ {}_1V = E[{}_1L^2 - 80\ {}_1L]. \tag{1}$$

Actually, we can calculate $E[{}_1L^2]$ and $E[{}_1L]$. Then equation (1) becomes a second order equation for ${}_1V$. We can solve the equation to obtain ${}_1V$. Similar idea can be applied when we calculate ${}_2V,\ {}_3V,\ {}_4V$.

4. (7.31)(a) Let P be the annual premiums collected from age 35 to age 64. According to the assumptions, we have

$$L = \begin{cases} \nu^{K+1} - P_{25}\ddot{a}_{\overline{K+1}|}, & K < 10; \\ \nu^{K+1} - P_{25}\ddot{a}_{\overline{10}|} - P(\ddot{a}_{\overline{K+1}|} - \ddot{a}_{\overline{10}|}), & 10 \leq K < 40; \\ \nu^{K+1} - P_{25}\ddot{a}_{\overline{10}|} - P(\ddot{a}_{\overline{40}|} - \ddot{a}_{\overline{10}|}), & K \geq 40. \end{cases}$$

It is not hard to verify that

$$E[L] = A_{25} - P_{25}\ddot{a}_{25:\overline{10}|} - P(\ddot{a}_{25:\overline{40}|} - \ddot{a}_{25:\overline{10}|})$$

Set $E[L] = 0$, and we can obtain that

$$P = \frac{A_{25} - P_{25}\ddot{a}_{25:\overline{10}|}}{\ddot{a}_{25:\overline{40}|} - \ddot{a}_{25:\overline{10}|}}. \tag{2}$$

From the Illustrative Life table on page 678, we can get

$$A_{25} = 0.0816496, \quad \ddot{a}_{25} = 16.22419. \tag{3}$$

Therefore, we have

$$P_{25} = \frac{A_{25}}{\ddot{a}_{25}} = \frac{0.0816496}{16.22419} = 0.0050326. \tag{4}$$

Next we need to calculate $\ddot{a}_{25:\overline{40}|}, \ddot{a}_{25:\overline{10}|}$. We will use the formula

$$\ddot{a}_{x:\overline{n}|} = \frac{1 - A_{x:\overline{n}|}}{d}. \tag{5}$$

Recall that

$$A_x = A^1_{x:\overline{n}|} + A_{x+n} A_{x:\overset{1}{\overline{n}|}},$$

and we can get that

$$A_{x:\overline{n}|} = A^1_{x:\overline{n}|} + A_{x:\overset{1}{\overline{n}|}} = A_x + (1 - A_{x+n}) A_{x:\overset{1}{\overline{n}|}}.$$

In addition, we have

$$A_{x:\overset{1}{\overline{n}|}} = \nu^n \cdot {}_n p_x = (1+i)^{-n} \cdot \frac{l_{x+n}}{l_x},$$

where l_{x+n}, l_x can be obtained from the life table. Therefore, we can have

$$A_{x:\overline{n}|} = A_x + \frac{(1 - A_{x+n}) l_{x+n}}{(1+i)^n l_x}. \tag{6}$$

After we get the value of $A_x, A_{x+n}, l_x, l_{x+n}$ from the life table, we are able to calculate $\ddot{a}_{x:\overline{n}|}$ by virtue of $(6), (5)$. Using the life table at page 675-680, we can get that

$$\ddot{a}_{25:\overline{40}|} = 15.4663, \quad \ddot{a}_{25:\overline{10}|} = 7.758. \tag{7}$$

Then, by virtue of $(2), (3), (4)$ and (7), we have

$$P = 0.00553.$$

(b) Let K be the curtate future life time for (35), and define

$$Y = \begin{cases} \ddot{a}_{\overline{K+1}|}, & K < 30; \\ \ddot{a}_{\overline{30}|}, & K \geq 30. \end{cases}$$

Given above, we can get

$${}_{10}L = \nu^{K+1} - PY,$$

where P is given in (a). Therefore, we have

$${}_{10}V = E[{}_{10}L] = A_{35} - P\ddot{a}_{35:\overline{30}|}.$$

Similar to what we did in (a), we can obtain $A_{35}, \ddot{a}_{35:\overline{30}|}$ by virtue of the life table. Finally, we can get

$${}_{10}V = 0.05126.$$

(c) Let K be the curtate future life time for (35). According to the assumption, we have

$$ {}_{10}L = B\nu^{K+1} - P_{25}Y, $$

where Y is as given in (b). Set $E[{}_{10}L] = 0$ and we can get

$$ B = \frac{P_{25}\ddot{a}_{35:\overline{30}|}}{A_{35}}. $$

By virtue of the life table, we can obtain that $B = 0.94612$.

(d) Let K be the curtate future life time for (45). Define Y_1 as

$$ Y_1 = \begin{cases} \ddot{a}_{\overline{K+1}|}, & K < 20; \\ \ddot{a}_{\overline{20}|}, & K \geq 20. \end{cases} $$

Then we have

$$ {}_{20}L = B\nu^{K+1} - P_{25}Y_1. $$

So we can get that

$$ {}_{20}V = E[{}_{20}L] = BA_{45} - P_{25}\ddot{a}_{45:\overline{20}|}. $$

By virtue of the life table, we can obtain that ${}_{20}V = 0.13211$.