MA (ST) 412 Supplement to 6.3
Solutions

1. By definition, we have

$$A_{30} = \sum_{k=0}^{69} \nu^{k+1} \cdot \frac{1}{70} = \frac{1}{70} \cdot \frac{\nu - \nu^{71}}{1 - \nu} = 0.2639$$

Therefore, we can get

$$P_{30} = \frac{dA_{30}}{1 - A_{30}} = 0.01793.$$

On the other hand, we have

$$L = \left(1 + \frac{P_{30}}{d}\right) \nu^{k+1} - \frac{P_{30}}{d} = 1.3586\nu^{K+1} - 0.3586.$$

Therefore, we can have

$$\begin{aligned} Pr(L \leq 0) &= Pr(1.3586\nu^{K+1} - 0.3586 \leq 0) \\ &= Pr(\nu^{K+1} \leq 0.2639) \\ &= Pr(K \geq \frac{\ln 0.2639}{\ln \nu} - 1) \\ &= Pr(K \geq 25) = \frac{s(55)}{s(30)} = 0.6429. \end{aligned}$$

2. According to the assumptions, we have $\nu = 1 - d = 0.96$ and

$$L = \nu^{K+1} - 0.05\ddot{a}_{\overline{K+1}|} = \nu^{K+1} - 0.05 \cdot \frac{1 - \nu^{K+1}}{d} = 2.25\nu^{K+1} - 1.25.$$

Therefore,

$$E[L] = 2.25E[\nu^{K+1}] - 1.25 = 2.25A_x - 1.25 = -0.125,$$
$$Var(L) = 2.25^2 \cdot Var(\nu^{K+1}) = 2.25^2 * [({}^2A_x) - (A_x)^2] = 0.2531.$$

3. According to Problem 2, we have

$$E[L] = -0.125, \quad Var(L) = 0.2531.$$

Let S be the total loss-at-issue. Then, we have

$$E[S] = 135 \cdot E[L] + 10E[3L] = 165E[L] = -20.625,$$
$$Var(S) = 135Var(L) + 10Var(3L) = 225Var(L) = 56.948.$$

Using Normal Distribution table, we have

$$\begin{aligned} Pr(-S > 45) &= Pr(S < -45) = Pr\left(\frac{S - E[S]}{\sqrt{Var(S)}} < \frac{-45 - E[S]}{\sqrt{Var(S)}}\right) \\ &= Pr\left(\frac{S - E[S]}{\sqrt{Var(S)}} < -3.23\right) = 0.0007. \end{aligned}$$