MA (ST) 412 Supplement to 5.3
Solutions

1.
$$d = \frac{i}{1+i} = 0.04, \quad e^{-\delta} = 1 - d = 0.96.$$

Therefore,
$$^2d = 1 - e^{-2\delta} = (1 + e^{-\delta})(1 - e^{-\delta}) = 0.0784.$$

Given above, we can get
$$A_x = 1 - d\ddot{a}_x = 0.6, \quad {}^2A_x = 1 - {}^2d \cdot {}^2\ddot{a}_x = 0.5296.$$

Thus,
$$Var(Y) = \frac{{}^2A_x - (A_x)^2}{d^2} = 106.$$

2. You can verify
$$_{k|}q_x = Pr(K = k).$$

Therefore, according to (5.3.9) on page 144 of *Actuarial Mathematics*, we have
$$\begin{aligned}\ddot{a}_{x:\overline{4|}} &= \sum_{k=0}^{3} \ddot{a}_{\overline{k+1|}} Pr(K = k) + \ddot{a}_{\overline{4|}} \cdot {}_4p_x \\ &= \sum_{k=0}^{3} \ddot{a}_{\overline{k+1|}} \cdot {}_{k|}q_x + \ddot{a}_{\overline{4|}} \left(1 - \sum_{k=0}^{3} {}_{k|}q_x\right) \\ &= 2.2186.\end{aligned}$$

3. From the given data, we can get
$$T = 30.7 - 25 = 5.7, \quad K = 5.$$

Therefore, we have the following results:

(a) $\ddot{a}_{\overline{K+1|}} = \ddot{a}_{\overline{6|}}$,

(b) $\ddot{a}_{\overline{5|}}$,

(c) $\ddot{a}_{\overline{K+1|}} - \ddot{a}_{\overline{2|}} = \ddot{a}_{\overline{6|}} - \ddot{a}_{\overline{2|}}$,

(d) $\ddot{a}_{\overline{10}|}$.

4. From the assumption, we know that the survival function is given by

$$s(x) = 1 - \frac{x}{100}, \quad 0 \leq x \leq 100.$$

Therefore, we have

$$ {}_kp_x = \frac{s(x+k)}{s(x)} = \frac{1 - \frac{x+k}{100}}{1 - \frac{x}{100}} = 1 - \frac{k}{100-x}, \quad 0 \leq k \leq 100 - x.$$

On the other hand, since $i = 0$, we have $\nu = \frac{1}{1+i} = 1$.

(a)

$$ {}_{10|}\ddot{a}_{50} = \sum_{k=10}^{50} \nu^k {}_kp_{50} = \sum_{k=10}^{50} \left(1 - \frac{k}{50}\right) = 16.4.$$

(b) The actual payments will start at age 60, 10 years later and will stop at time $K+1$, the end of the year of death. Thus, the present value of the actual payments is $\ddot{a}_{\overline{K+1}|} - \ddot{a}_{\overline{10}|}$. The probability we want is

$$\begin{aligned} Pr(\ddot{a}_{\overline{K+1}|} - \ddot{a}_{\overline{10}|} > {}_{10|}\ddot{a}_{50}) &= Pr(K + 1 - 10 > 16.4) \\ &= Pr(K \geq 26) \\ &= 1 - \frac{26}{100-50} \\ &= 0.48. \end{aligned}$$