

MA 732 Supplementary Problems

February 6, 2004

1. Suppose $f : X \rightarrow Y$ satisfies (1) f is differentiable at x , and (2) for all x' in some neighborhood of x , $\|f(x') - f(x)\| \leq \lambda\|x' - x\|$. Show that $\|Df(x)\| \leq \lambda$.
2. Let $A(t)$ be a 3×3 matrix depending differentiably on t , and let $f(t)$ and $g(t)$ be 3×1 vectors depending differentiably on t . Compute $\frac{d}{dt}(A(t)(f(t) \times g(t)))$ using the generalized product rule. (Here \times is cross product.)
3. Consider the map $F : C^0([a, b], R^n) \rightarrow R$, $F(\phi) = \int_a^b \phi(t) \cdot \phi(t) dt$. Write F as a composition of *three* maps, each of which is linear or bilinear. Then use the chain rule and generalized product rule to compute $DF(\phi)$.