MA 532 Homework 1

S. Schecter

August 22, 2008

1. Sketch the phase portrait.

   (a) $\dot{x} = x - x^3$

   (b) $\dot{x} = 1 + x^2$

   (c) $\dot{x} = x^2 - x^3$

   (d) $\dot{x} = 1 - \cos x$

2. The population of fish in a lake satisfies the differential equation

   $$\dot{x} = ax - bx^2,$$

   where x is the number of fish, t is time in years, $a > 0$ and $b > 0$ are constants. The manager of the lake proposes to allow fishing at a rate of h fish per year. Complete the following sentence: If h is greater than [fill in the blank with a number that depends on a and b], the fish population will crash. Justify your answer using phase portraits.

   Suggestion: The new differential equation is

   $$\dot{x} = ax - bx^2 - h.$$

   The phase portrait depend on the number of roots of the equation $ax - bx^2 - h = 0$.