

# MA 532 Supplementary Problems 5

October 13, 2003

1. Let  $C = \begin{pmatrix} 0 & -\beta \\ \beta & 0 \end{pmatrix}$ .

(a) Show that  $C^{2k} = (-1)^k \begin{pmatrix} \beta^{2k} & 0 \\ 0 & \beta^{2k} \end{pmatrix}$ .

(b) Show that  $C^{2k+1} = (-1)^k \begin{pmatrix} 0 & -\beta^{2k+1} \\ \beta^{2k+1} & 0 \end{pmatrix}$ .

(c) By adding an infinite series, show that  $e^{tC} = \begin{pmatrix} \cos \beta t & -\sin \beta t \\ \sin \beta t & \cos \beta t \end{pmatrix}$ .

2. Show that if  $J = \begin{pmatrix} \lambda & 1 & 0 \\ 0 & \lambda & 1 \\ 0 & 0 & \lambda \end{pmatrix}$ , then  $e^{tJ} = e^{t\lambda} \begin{pmatrix} 1 & t & \frac{1}{2}t^2 \\ 0 & 1 & t \\ 0 & 0 & 1 \end{pmatrix}$ .

3. Find  $e^{tA}$ .

(a)  $A = \begin{pmatrix} 6 & -3 \\ 2 & 1 \end{pmatrix}$ .

(b)  $A = \begin{pmatrix} -4 & 2 \\ 2 & -1 \end{pmatrix}$ .

(c)  $A = \begin{pmatrix} -1 & -2 & 2 \\ -2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$ .

(d)  $A = \begin{pmatrix} -1 & 2 \\ -1 & -3 \end{pmatrix}$ .

(e)  $A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$ .

(f)  $A = \begin{pmatrix} 1 & -1 \\ 1 & 3 \end{pmatrix}$ .

(g)  $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -3 & -3 \end{pmatrix}$ .