1. Consider the differential equation $\ddot{x} - x + x^3 = 0$.
   
   (a) Convert to a system by letting $y = \dot{x}$.
   (b) Find the equilibria of the system.
   (c) Draw the phase portrait of the system.

2. Consider the differential equation $\ddot{x} + \epsilon \dot{x} - x + x^3 = 0$, $\epsilon > 0$. (We have added damping to Problem 1. Your work on Problem 1 will help you do this one.)
   
   (a) Convert to a system by letting $y = \dot{x}$.
   (b) Use a Lyapunov function suggested by your work on Problem 1, together with Lasalle’s Invariance Principle, to prove that every solution approaches one of the equilibria as $t \to \infty$.

3. Consider the differential equation $\ddot{x} + \dot{x} + x - x^3 = 0$. (We have added damping to Exercise 1.36 (3) on p. 27. Your work on that problem will help you do this one.)
   
   (a) Convert to a system by letting $y = \dot{x}$.
   (b) Define an open set $G$ in $\mathbb{R}^2$ such that if $p \in G$, then the solution through $p$ approaches the origin as $t \to \infty$.
   (c) Use a Lyapunov function suggested by your work on Exercise 1.36 (3), together with Lasalle’s Invariance Principle, to prove that if $p \in G$, then the solution through $p$ approaches the origin as $t \to \infty$.

4. Let $\dot{x} = f(x)$ be a $C^1$ differential equation on $\mathbb{R}^n$, and let $p \in \mathbb{R}^n$. Let $\Gamma = \{ \varphi(t, p) : t \geq 0 \}$, the positive semi-orbit through $p$. We use $\text{cl}(A)$ to denote the closure of a set $A$.
   
   (a) Prove: $\text{cl}(\Gamma) = \Gamma \cup \omega(p)$.
   (b) Prove: If $\omega(p)$ is empty, then $\Gamma$ is closed.
   (c) Prove: If $\Gamma$ is compact, then $\Gamma = \omega(p)$. 
