MA 493 Homework 7

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1. Consider the game "Reporting a Crime" in section 8.2 of the on-line notes.

   (a) Suppose players 1 and 2 use the same mixed strategy $(1-p)C+pN$, with $0 < p < 1$, and all other players use the strategy N. Find a value of p for which this is a Nash equilibrium.

   (b) In the Nash equilibrium of part (a), what is the probability that the police get called?

   (c) Extra credit: Let k be a number between 1 and n. Suppose players $1, \ldots, k$ use the same mixed strategy $(1-p)C + pN$, with $0 < p < 1$, and all other players use the strategy N. Find a value of p for which this is a Nash equilibrium. In this Nash equilibrium, what is the probability that the police get called?

2. This problem is based on a scene in the movie "A Beautiful Mind." n men walk into a bar. In the bar is one extremely attractive woman and many attractive women. Each man has two possible pure strategies:

   - S: Approach one of the attractive women. (The safe strategy.)
   - R: Approach the extremely attractive woman. (The risky strategy.)

   The payoffs are:

   - $a > 0$ to each man who uses strategy S. (There are many attractive women in the bar; the strategy of approaching one of them will succeed.)

- If there is a unique man who uses strategy R, his payoff is $b > a$. If more than one man uses strategy R, they all have payoff 0. (The extremely attractive woman doesn't enjoy being pestered and leaves.)

(a) Find all pure strategy Nash equilibria of this n-person game.

(b) Find a mixed strategy Nash equilibrium in which all n men use the same mixed strategy $pS + (1 - p)R$.

(c) In the Nash equilibrium of part (b), for large n, what is the approximate probability that at least one man approaches the extremely attractive woman?

3. Problem 6.9 in Gintis. The answer to part a is given below, but you should explain the numbers. In part c, ignore the last four questions.

		Iraq	
		2	**4**
Iran	**2**	(46,42)	(26,44)
	4	(52,22)	(32,24)

4. Problem 6.12 in Gintis. In this problem the ith student's strategy is a number x_i, $0 \leq x_i \leq 1$, which represents the amount that student chooses to deposit in the public account. A strategy profile is therefore a 10-tuple $(x_1, x_2, \ldots, x_{10})$ with $0 \leq x_i \leq 1$ for each i.

   Ignore questions a–d, and instead answer the following.

   (a) Find the ith player's payoff function $\Pi_i(x_1, x_2, \ldots, x_{10})$. (The answer is $\Pi_i(x_1, x_2, \ldots, x_{10}) = 1 - x_i + \frac{1}{2}(x_1 + x_2 + \ldots + x_{10})$; you should explain this.)

   (b) Show that each player has a strictly dominant strategy: Whatever the other players do, contribute nothing. (For example, consider player 1. Given any choices $(x_2, \ldots, x_{10})$ by the other players, player 1 maximizes his payoff by choosing $x_1 = 0$.) Therefore the only Nash equilibrium is $(0, 0, \ldots, 0)$, at which each player's payoff is 1.

(c) Suppose the game is repeated every day. Consider the following strategy σ_x, where $0 < x \leq 1$: "I will contribute x dollars on day 0. If every other player contributes at least x dollars on day k, I will contribute x dollars on day $k+1$. If any player contributes less than x dollars on day k, I will contribute nothing on every subsequent day." Show that if $\delta \geq \frac{1}{9}$, then it is a Nash equilibrium for every player to use the strategy σ_x with the same x. (In other words, $(\sigma_x, \sigma_x, \ldots, \sigma_x)$ is a Nash equilibrium.)