

MA 493 Homework 6

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1. Gintis, problem 4.14. Assume $c_b \neq 2c_\ell$. In part (a), for each pair of pure strategies, find conditions under which that pair can be a Nash equilibrium, or state that that pair can never be a Nash equilibrium. (For example, you should find that (b, b) is a Nash equilibrium if $c_b > 2c_\ell$.) Do parts (b) and (c) as stated.
2. Gintis, problem 4.25. Player 1 has three strategies (pick 1, pick 2, pick 3). Player 2 has five strategies:
 - (1) Guess 1. If told it is low, guess 2.
 - (2) Guess 1. If told it is low, guess 3.
 - (3) Guess 2. If told it is high, guess 1. If told it is low, guess 2.
 - (4) Guess 3. If told it is high, guess 1.
 - (5) Guess 3. If told it is high, guess 2.

(a) For part (a), just do the following.

 - i. Construct the payoff matrix, and check your work on p. 420.
 - ii. Show that there are no pure strategy Nash equilibria.
 - iii. To look for mixed strategy Nash equilibria, let $\sigma_1 = (p_1, p_2, p_3)$ be a mixed strategy for player 1, and let $\sigma_2 = (q_1, q_2, q_3, q_4, q_5)$ be a mixed strategy for player 2. Find a Nash equilibrium in which all player 1's strategies are active, and only player 2's second, third, and fourth strategies are active.
 - iv. Determine whether there is a Nash equilibrium in which all player 1's strategies are active, and only player 2's first, third, and fifth strategies are active.

- (b) For part (b), find the expected payoff to player 2 from the mixed strategy Nash equilibrium you found in part (a)(iii).
3. Gintis, problem 4.31 (a) and (c). To begin, ignore the sentence that says “We can normalize . . .,” and derive the payoff matrix, explaining each entry. Your matrix should look just like the one in the text, except that the payoff to player 1 in the lower right box will be $(1 - \alpha)b - \alpha p = 1 - \alpha(b + p)$. The matrix in the book is obtained by taking your matrix, dividing all of player 1’s payoffs by $b + p$, replacing each term $\frac{b}{b+p}$ in the resulting expressions by a new variable, say b' , and then “simplifying” by changing b' to b . Now do parts (a) and (c) using the payoff matrix in the text.
4. Problem 4.23. Do the following subproblems instead of those suggested in the text.
- (a) Each firm has two strategies, M and E . Give the payoffs to each triple of pure strategies. You should organize your answer by giving two 2×2 matrices; see section 3.9 of the online notes.
- (b) Show that every triple of pure strategies except (M, M, M) and (E, E, E) is a Nash equilibrium.
- (c) Suppose firm 1 uses the mixed strategy $xM + (1 - x)E$, firm 2 uses the mixed strategy $yM + (1 - y)E$, and firm 3 uses the mixed strategy $zM + (1 - z)E$. Show that the payoff functions are

$$\begin{aligned}\Pi_1(x, y, z) &= 2(1 - x)yz + x(1 - y)(1 - z), \\ \Pi_2(x, y, z) &= 2(1 - y)xz + y(1 - x)(1 - z), \\ \Pi_3(x, y, z) &= 2(1 - z)xy + z(1 - x)(1 - y).\end{aligned}$$

- (d) Suppose one player uses the pure strategy M and one uses the pure strategy E . Show that any mix of strategies by the third player yields a Nash equilibrium. (For example, for any z with $0 \leq z \leq 1$, $(M, E, zM + (1 - z)E)$ is a Nash equilibrium.)
- (e) Show that there is no Nash equilibrium in which exactly one firm uses a pure strategy. Hint: Suppose firm 3 uses the pure strategy M , so that $z = 1$. If there is a Nash equilibrium with $0 < x < 1$ and $0 < y < 1$, then we must have

$$\frac{\partial \Pi_1}{\partial x}(x, y, 1) = 0 \text{ and } \frac{\partial \Pi_2}{\partial y}(x, y, 1) = 0.$$

(f) Find a Nash equilibrium in which no firm uses a pure strategy.