1. Gintis, problem 4.14. Assume $c_b \neq 2 c_\ell$. In part (a), for each pair of pure strategies, find conditions under which that pair can be a Nash equilibrium, or state that that pair can never be a Nash equilibrium. (For example, you should find that $(b, b)$ is a Nash equilibrium if $c_b > 2 c_\ell$.) Do parts (b) and (c) as stated.

2. Gintis, problem 4.25. Player 1 has three strategies (pick 1, pick 2, pick 3). Player 2 has five strategies:
   
   (1) Guess 1. If told it is low, guess 2.
   (2) Guess 1. If told it is low, guess 3.
   (3) Guess 2. If told it is high, guess 1. If told it is low, guess 2.
   (4) Guess 3. If told it is high, guess 1.
   (5) Guess 3. If told it is high, guess 2.

(a) For part (a), just do the following.
   
   i. Construct the payoff matrix, and check your work on p. 420.
   ii. Show that there are no pure strategy Nash equilibria.
   iii. To look for mixed strategy Nash equilibria, let $\sigma_1 = (p_1, p_2, p_3)$ be a mixed strategy for player 1, and let $\sigma_2 = (q_1, q_2, q_3, q_4, q_5)$ be a mixed strategy for player 2. Find a Nash equilibrium in which all player 1’s strategies are active, and only player 2’s second, third, and fourth strategies are active.
   iv. Determine whether there is a Nash equilibrium in which all player 1’s strategies are active, and only player 2’s first, third, and fifth strategies are active.
(b) For part (b), find the expected payoff to player 2 from the mixed
strategy Nash equilibrium you found in part (a)(iii).

3. Gintis, problem 4.31 (a) and (c). To begin, ignore the sentence that
says “We can normalize . . . ,” and derive the payoff matrix, explaining
each entry. Your matrix should look just like the one in the text, except
that the payoff to player 1 in the lower right box will be \((1 - \alpha)b - \alpha p = 1 - \alpha(b + p)\). The matrix in the book is obtained by taking your matrix,
dividing all of player 1’s payoffs by \(b + p\), replacing each term \(\frac{b}{b+p}\) in the
resulting expressions by a new variable, say \(b'\), and then “simplifying”
by changing \(b'\) to \(b\). Now do parts (a) and (c) using the payoff matrix
in the text.

4. Problem 4.23. Do the following subproblems instead of those suggested
in the text.

(a) Each firm has two strategies, \(M\) and \(E\). Give the payoffs to each
triple of pure strategies. You should organize your answer by
giving two \(2 \times 2\) matrices; see section 3.9 of the online notes.
(b) Show that every triple of pure strategies except \((M, M, M)\) and
\((E, E, E)\) is a Nash equilibrium.
(c) Suppose firm 1 uses the mixed strategy \(xM + (1-x)E\), firm 2
uses the mixed strategy \(yM + (1-y)E\), and firm 3 uses the mixed
strategy \(zM + (1-z)E\). Show that the payoff functions are
\[
\begin{align*}
\Pi_1(x, y, z) &= 2(1-x)yz + x(1-y)(1-z), \\
\Pi_2(x, y, z) &= 2(1-y)zx + y(1-x)(1-z), \\
\Pi_3(x, y, z) &= 2(1-z)xy + z(1-x)(1-y).
\end{align*}
\]
(d) Suppose one player uses the pure strategy \(M\) and one uses the
pure strategy \(E\). Show that any mix of strategies by the third
player yields a Nash equilibrium. (For example, for any \(z\) with
\(0 \leq z \leq 1\), \((M, E, zM + (1-z)E)\) is a Nash equilibrium.)
(e) Show that there is no Nash equilibrium in which exactly one firm
uses a pure strategy. Hint: Suppose firm 3 uses the pure strategy
\(M\), so that \(z = 1\). If there is a Nash equilibrium with \(0 < x < 1\)
and \(0 < y < 1\), then we must have
\[
\frac{\partial \Pi_1}{\partial x}(x, y, 1) = 0 \quad \text{and} \quad \frac{\partial \Pi_2}{\partial y}(x, y, 1) = 0.
\]
(f) Find a Nash equilibrium in which no firm uses a pure strategy.