

MA 493 Homework 3

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This problem set is based on problem 2.11 in the text, but I've made it a little more precise.

A Ming vase is sold at auction. The auction works like this. The auctioneer calls out the price k dollars. Any bidder who wants may raise her hand.

1. If more than one bidder raises her hand, the auctioneer calls out the price $k + 1$ dollars.
2. If exactly one bidder raises her hand, the auction is over, and the vase is sold to that bidder for k dollars.
3. If no bidder raises her hand, the auction is over, but the vase is not sold to anyone.

The bidders raise their hands simultaneously. The auctioneer begins at 1 dollar.

A strategy for bidder i is simply the set of prices at which bidder i will raise her hand, if the auctioneer calls out that price. For example, if bidder i is willing to bid up to 5 dollars, her strategy is the set $\{1, 2, 3, 4, 5\}$.

You may assume that each bidder's strategy is a finite set. However, you should not assume that each bidder's strategy is a set of consecutive integers that starts with 1. For example, a possible strategy is $\{2, 4, 5\}$.

The value of the vase to the i th bidder is v_i dollars; v_i is a positive integer.

1. Let s_i be a strategy for player i in which the highest bid is k dollars, with $k > v_i$. Let t_i be the strategy for player i that is obtained from s_i by deleting the bid k . Show that t_i weakly dominates s_i . (Suggestion: For any choice of strategies by the other players, if the auction is over

before the bidding reaches k dollars, t_i and s_i give the same result. What if the bidding reaches k dollars?)

2. Explain why problem 1 implies that every strategy of player i in which some bid is higher than v_i dollars is weakly dominated by a strategy in which no bid is higher than v_i dollars.
3. Let s_i be a strategy for player i in which the highest bid is k dollars, with $k \leq v_i$. Suppose s_i does not include all bids from 1 to k . Let ℓ be the lowest bid that is not included in s_i . Let t_i be the strategy for player i obtained from s_i by including the bid ℓ . Show that t_i weakly dominates s_i . (Suggestion: Consider the following cases: (1) The auction is over before the bidding reaches ℓ dollars. (2) The auction reaches ℓ dollars, but no bidder other than the i th bids ℓ dollars. (3) The auction reaches ℓ dollars, and exactly one bidder other than the i th bids ℓ dollars. (4) The auction reaches ℓ dollars, and two or more bidders other than the i th bid ℓ dollars.)
4. Consider the following collection of strategies for player i :

$$\{1\}, \{1, 2\}, \{1, 2, 3\}, \dots, \{1, 2, \dots, v_i\}.$$

Explain why problems 1–3 imply that every strategy for player i that is not in this collection is weakly dominated by one of the strategies in the collection.

5. Show that player i 's strategies $\{1\}, \{1, 2\}, \{1, 2, 3\}, \dots, \{1, 2, \dots, v_i - 2\}$ are all weakly dominated by her strategies $\{1, 2, \dots, v_i - 1\}$ and $\{1, 2, \dots, v_i\}$.
6. Does either of player i 's strategies $\{1, 2, \dots, v_i - 1\}$ and $\{1, 2, \dots, v_i\}$ weakly dominate the other?