

# MA 493B Homework

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1. Problem 9.17. The payoff matrix is in the text. Consider this as an asymmetric evolutionary game, and analyze it as we did Big Monkey and Little Monkey:
  - (a) Check for pure strategy Nash equilibria.
  - (b) Assume the buyer uses strategy  $I$  with probability  $p$  and strategy  $T$  with probability  $1 - p$ , and the seller uses strategy  $H$  with probability  $q$  and strategy  $D$  with probability  $1 - q$ . Derive the replicator equation ( $\dot{p}$  and  $\dot{q}$  equations only).
  - (c) Find all equilibria of the replicator equation in the region  $0 \leq p \leq 1$ ,  $0 \leq q \leq 1$ .
  - (d) Find where  $\dot{p}$  and  $\dot{q}$  are 0, and use this information to sketch the vector field in the  $0 \leq p \leq 1$ ,  $0 \leq q \leq 1$ . Be sure to show the direction of flow on the boundary of the region. One cannot tell from the vector field whether there are closed orbits.
  - (e) Find the eigenvalues of the linearization at the equilibrium  $(\frac{1}{2}, \frac{1}{2})$ . They don't help.
  - (f) Using separation of variables, show that the function  $f(p, q) = (p - p^2)(q - q^2)$  is constant on solutions. Does this help you decide whether there are closed orbits?

The evolution of cooperative social behavior is a topic of current interest in biology. For a recent review article, see Nature, 27 October 2005 (<http://www.nature.com/nature/journal/v437/n7063/pdf/nature04131.pdf>).