

MA 493 Game Theory Homework

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First, the definition I gave in class of a weakly dominated strategy was not correct. The correct definition is on p. 15 of our text: Suppose s_i and t_i are two strategies for player i . We say s_i is weakly dominated by t_i if for every choice of strategies by the other players, the payoff to player i from using t_i is greater than or equal to the payoff to player i from using s_i . (It is not necessary that for some choice of strategies by the other players, the payoff to player i from t_i be greater than the payoff to player i from s_i .)

This problem is based on problem 2.11 in the text, but I've made it a little more precise.

A Ming vase is sold at auction. The auction works like this. The auctioneer calls out the price k dollars. Any bidder who wants may raise his hand.

1. If more than one bidder raises his hand, the auctioneer calls out the price $k + 1$ dollars.
2. If exactly one bidder raises his hand, the auction is over, and the vase is sold to that bidder for k dollars.
3. If no bidder raises his hand, the auction is over, but the vase is not sold to anyone.

The bidders raise their hands simultaneously. The auctioneer begins at 1 dollar.

A strategy for bidder i is simply the set of prices at which bidder i will raise his hand, if the auctioneer calls out that price. For example, if bidder i is willing to bid up to 5 dollars, his strategy is the set $\{1, 2, 3, 4, 5\}$.

You may assume that each bidder's strategy is a finite set. However, you should *not* assume that each bidder's strategy is a set of consecutive integers that starts with 1. For example, a possible strategy is $\{2, 4, 5\}$.

The value of the vase to the i th bidder is v_i dollars; v_i is a positive integer.

1. Let s_i be a strategy for player i in which the highest bid is k dollars, with $k > v_i$. Let t_i be the strategy for player i that is obtained from s_i by deleting the bid k . Show that s_i is weakly dominated by t_i . (Suggestion: For any choice of strategies by the other players, if the auction is over before the bidding reaches k dollars, s_i and t_i give the same result. If the bidding gets to k dollars, s_i is sure to yield a payoff to player i that is 0 or negative. What about t_i ?)
2. Let s_i be a strategy for player i in which the highest bid is k dollars, with $k < v_i$. Let t_i be the strategy for player i obtained from s_i by including the bid v_i . Show that s_i is weakly dominated by t_i . (Remember the corrected definition of weakly dominated given above.)
3. Let s_i be a strategy for player i in which the highest bid is v_i dollars, but s_i does not include all bids from 1 to v_i . Let k be the lowest bid that is not included in s_i . Let t_i be the strategy for player i obtained from s_i by including the bid k . Show that s_i is weakly dominated by t_i . (Suggestion: Consider the following cases: (1) The auction is over before the bidding reaches k dollars. (2) The auction reaches k dollars, but no bidder other than the i th bids k dollars. (3) The auction reaches k dollars, and exactly one bidder other than the i th bids k dollars. (4) The auction reaches k dollars, and two or more bidders other than the i th bid k dollars.)
4. Use parts 1, 2, and 3 to show that player i 's strategy $\{1, 2, \dots, v_i\}$ weakly dominates all of player i 's other strategies.
5. Show that player i 's strategy $\{1, 2, \dots, v_i - 1\}$ weakly dominates $\{1, 2, \dots, v_i\}$.
6. Does 5 contradict 4?