# MA 493 Homework 5

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Problem 4.23. I am going to suggest a series of steps that is more detailed than what the text suggests.

There are two pages to this problem. Don't forget p. 2.

1. Each firm has two strategies, $M$ and $E$. Give the payoffs to each triple of pure strategies. You may want to organize your answer by giving two $2 \times 2$ matrices, as we did in lecture for problem 4.20.

2. Suppose firm 1 uses the mixed strategy $xM + (1-x)E$, firm 2 uses the mixed strategy $yM + (1-y)E$, and firm 3 uses the mixed strategy $zM + (1-z)E$. Show that the payoff functions are

$$\begin{aligned}
\Pi_1(x, y, z) &= 2(1-x)yz + x(1-y)(1-z), \\
\Pi_2(x, y, z) &= 2(1-y)xz + y(1-x)(1-z), \\
\Pi_3(x, y, z) &= 2(1-z)xy + z(1-x)(1-y).
\end{aligned}$$

3. Show that there is no Nash equilibrium in which exactly one firm uses a pure strategy. Hint: Suppose firm 3 uses the pure strategy $M$, so that $z = 1$. If there is a Nash equilibrium with $0 < x < 1$ and $0 < y < 1$, then we must have

$$\frac{\partial \Pi_1}{\partial x}(x, y, 1) = 0 \text{ and } \frac{\partial \Pi_2}{\partial y}(x, y, 1) = 0.$$

4. Assume two players use the same pure strategy. Show that the only Nash equilibrium has the other player using the opposite pure strategy

5. Assume two players use different pure strategies (in other words, one uses $M$ and one uses $E$). Show that any mix of strategies by the third player yields a Nash equilibrium. (For example, for any $z$ with $0 \leq z \leq 1$, $(M, E, zM + (1-z)E)$ is a Nash equilibrium.)

6. Find a Nash equilibrium in which no firm uses a pure strategy.