

MA 493 Homework 4

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1. Problem 4.14. Assume $c_b \neq 2c_l$. In part (a), for each pair of pure strategies, find conditions under which that pair can be a Nash equilibrium, or state that that pair can never be a Nash equilibrium. (For example, you should find that (b, b) is a Nash equilibrium if $c_b > 2c_l$.) Do parts (b) and (c) as stated.
2. Problem 4.25. Player 1 has three strategies (pick 1, pick 2, pick 3). Player 2 has five strategies:
 - (1) Guess 1. If told it is low, guess 2.
 - (2) Guess 1. If told it is low, guess 3.
 - (3) Guess 2. If told it is high, guess 1. If told it is low, guess 2.
 - (4) Guess 3. If told it is high, guess 1.
 - (5) Guess 3. If told it is high, guess 2.
 - (a) For part (a), just do the following.
 - i. Construct the payoff matrix, and check your work on p. 420.
 - ii. Show that there are no pure strategy Nash equilibria.
 - iii. To look for mixed strategy Nash equilibria, let $\sigma_1 = (p_1, p_2, p_3)$ be a mixed strategy for player 1, and let $\sigma_2 = (q_1, q_2, q_3, q_4, q_5)$ be a mixed strategy for player 2. Show that there is no mixed strategy Nash equilibrium in which all q_i are positive. (Assuming all q_i are positive, derive a system of five equations that must be satisfied by (p_1, p_2, p_3) , and show that there are no solutions.)

- iv. Find a Nash equilibrium in which player 2's first and fifth strategies are not used (i.e., $q_1 = q_5 = 0$).
 - v. Determine whether there is a Nash equilibrium in which player 2's second and fourth strategies are not used (i.e., $q_2 = q_4 = 0$).
- (b) For part (b), find the expected payoff to player 2 from the mixed strategy Nash equilibrium you found in part (a)(iv).
3. Problem 4.31. To begin, ignore the sentence that says "We can normalize ...," and derive the payoff matrix, explaining each entry. Your matrix should look just like the one in the text, except that the payoff to player 1 in the lower right box will be $(1 - \alpha)b - \alpha p = 1 - \alpha(b + p)$. The matrix in the book is obtained by taking your matrix, dividing all of player 1's payoffs by $b + p$, replacing each term $\frac{b}{b+p}$ in the resulting expressions by a new variable, say b' , and then "simplifying" by changing b' to b .
- Now do parts (a), (b) and (c) using the payoff matrix in the text.