5.7 Huey, Dewey, and Louie Split a Dollar

S. Schecter

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Huey (player 1), Dewey (player 2), and Louie (player 3) have a dollar to split.

Round 0: Huey goes first and offers to split the dollar into fractions a_1 for himself, b_1 for Dewey, and c_1 for Louie, with $a_1 + b_1 + c_1 = 1$. If 3 both accept, the game is over. If at least one rejects the offer, the dollar shrinks to δ, and it is Dewey's turn to offer.

Round 1: Dewey (player 2) offers to split the dollar into fractions d_1 for Huey, e_1 for himself, and f_1 for Louie, with $d_1 + e_1 + f_1 = 1$. If Huey and Louie both accept, the game is over. If at least one rejects the offer, the dollar shrinks to δ^2, and it is Louie's turn to offer.

Round 2: Louie (player 3) offers to split the dollar into fractions g_1 for Huey, h_1 for Dewey, and k_1 for himself, with $g_1 + h_1 + k_1 = 1$. If Huey and Dewey both accept, the game is over. If at least one rejects the offer, the dollar shrinks to δ^3, and it is Huey's turn to offer.

Round 3: Huey (player 1) offers to split the dollar into fractions a_2 for himself, b_2 for Dewey, and c_2 for Louie, with $a_2 + b_2 + c_2 = 1$. If Dewey and Louie both accept, the game is over. If at least one rejects the offer, the dollar shrinks to δ^4, and it is Dewey's turn to offer.

Etc.

The game tree is shown on p.3.

Suppose we have a Nash equilibrium in which

1. $a_1 = a_2 = \ldots = a$;
2. $b_1 = b_2 = \ldots = b$;
3. $c_1 = c_2 = \ldots = c$;
4. $d_1 = d_2 = \ldots = d$;
5. $e_1 = e_2 = \ldots = e$;
6. $f_1 = f_2 = \ldots = f$;
7. $g_1 = g_2 = \ldots = g$;
8. $h_1 = h_2 = \ldots = h$;
9. $k_1 = k_2 = \ldots = k$;
10. whenever an offer is made, it is accepted.

Such a Nash equilibrium would be subgame perfect.

Questions:

1. By considering the offer at Round 0, show that $b = \delta e$ and $c = \delta f$.
2. By considering the offer at Round 1, show that $d = \delta g$ and $f = \delta k$.
3. By considering the offer at Round 2, show that $g = \delta a$ and $h = \delta b$.
4. From parts 1, 2, and 3 we have six equations in the nine variable $a, b, c, d, e, f, g, h, k$. We also have three more equations: $a + b + c = 1$, $d + e + f = 1$, $g + h + k = 1$. Show that the following is a solution of these nine equations. (Actually, it's the only solution.)

$$a = e = k = \frac{1}{1 + \delta + \delta^2}, \quad b = f = g = \frac{\delta}{1 + \delta + \delta^2}, \quad c = d = h = \frac{\delta^2}{1 + \delta + \delta^2}.$$

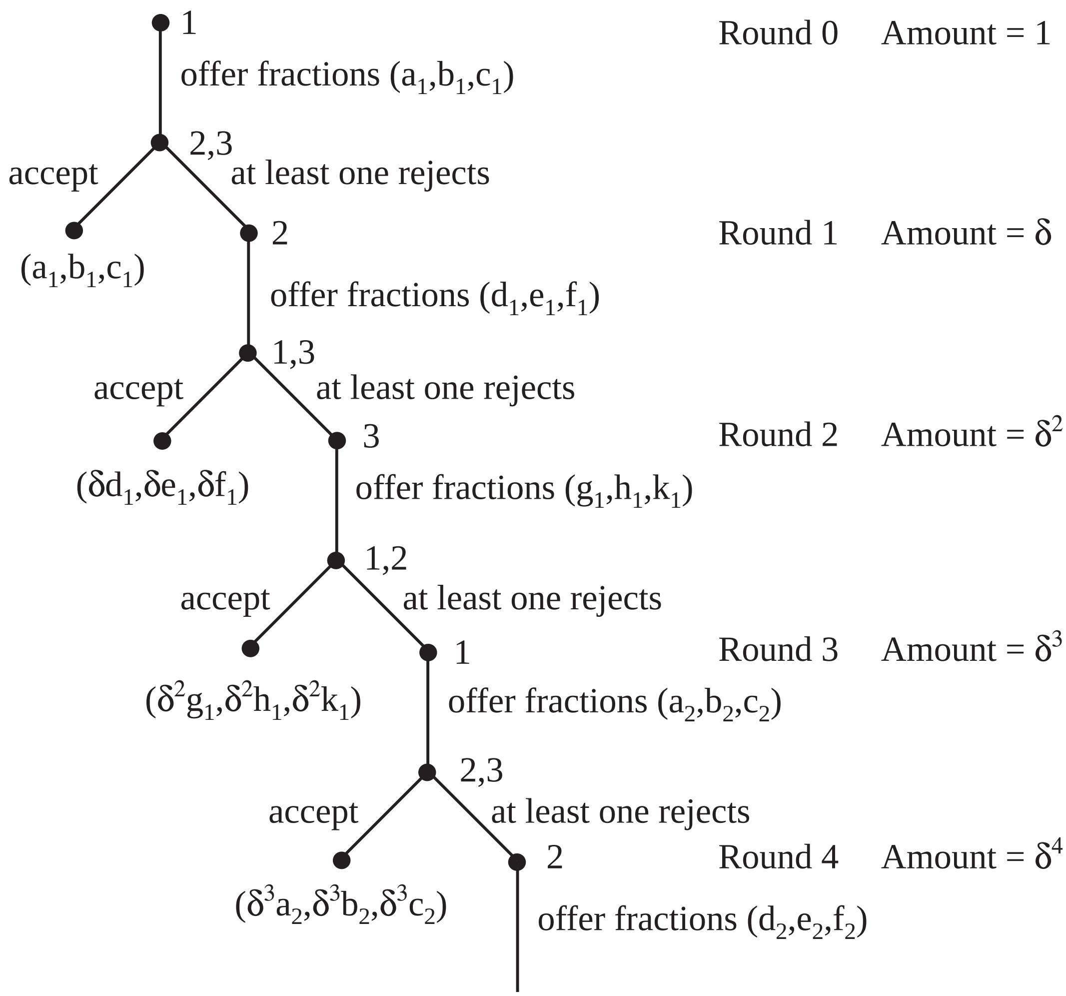

Figure 1: Game tree.