5.7 Huey, Dewey, and Louie Split a Dollar

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Huey (player 1), Dewey (player 2), and Louie (player 3) have a dollar to split.

Round 0: Huey goes first and offers to split the dollar into fractions $a_1$ for himself, $b_1$ for Dewey, and $c_1$ for Louie, with $a_1 + b_1 + c_1 = 1$. If 3 both accept, the game is over. If at least one rejects the offer, the dollar shrinks to $\delta$, and it is Dewey’s turn to offer.

Round 1: Dewey (player 2) offers to split the dollar into fractions $d_1$ for Huey, $e_1$ for himself, and $f_1$ for Louie, with $d_1 + e_1 + f_1 = 1$. If Huey and Louie both accept, the game is over. If at least one rejects the offer, the dollar shrinks to $\delta^2$, and it is Louie’s turn to offer.

Round 2: Louie (player 3) offers to split the dollar into fractions $g_1$ for Huey, $h_1$ for Dewey, and $k_1$ for himself, with $g_1 + h_1 + k_1 = 1$. If Huey and Dewey both accept, the game is over. If at least one rejects the offer, the dollar shrinks to $\delta^3$, and it is Huey’s turn to offer.

Round 3: Huey (player 1) offers to split the dollar into fractions $a_2$ for himself, $b_2$ for Dewey, and $c_2$ for Louie, with $a_2 + b_2 + c_2 = 1$. If Dewey and Louie both accept, the game is over. If at least one rejects the offer, the dollar shrinks to $\delta^4$, and it is Dewey’s turn to offer.

Etc.

The game tree is shown on p.3.
Suppose we have a Nash equilibrium in which

1. \(a_1 = a_2 = \ldots = a\);
2. \(b_1 = b_2 = \ldots = b\);
3. \(c_1 = c_2 = \ldots = c\);
4. \(d_1 = d_2 = \ldots = d\);
5. \(e_1 = e_2 = \ldots = e\);
6. \(f_1 = f_2 = \ldots = f\);
7. \(g_1 = g_2 = \ldots = g\);
8. \(h_1 = h_2 = \ldots = h\);
9. \(k_1 = k_2 = \ldots = k\);
10. whenever an offer is made, it is accepted.

Such a Nash equilibrium would be subgame perfect.

Questions:

1. By considering the offer at Round 0, show that \(b = \delta e\) and \(c = \delta f\).
2. By considering the offer at Round 1, show that \(d = \delta g\) and \(f = \delta k\).
3. By considering the offer at Round 2, show that \(g = \delta a\) and \(h = \delta b\).
4. From parts 1, 2, and 3 we have six equations in the nine variable \(a, b, c, d, e, f, g, h, k\). We also have three more equations: \(a + b + c = 1\), \(d + e + f = 1\), \(g + h + k = 1\). Show that the following is a solution of these nine equations. (Actually, it’s the only solution.)

\[
a = e = k = \frac{1}{1 + \delta + \delta^2}, \quad b = f = g = \frac{\delta}{1 + \delta + \delta^2}, \quad c = d = h = \frac{\delta^2}{1 + \delta + \delta^2}.
\]
Figure 1: Game tree.