1. Consider the game of lions and antelopes (6.2 in Gintis) in the case $c_b = 2c_l$. For definiteness we’ll take $c_b = 4$ and $c_l = 2$. The payoff matrix is then

\[
\begin{array}{c|cc}
& \text{b} & \text{l} \\
\hline
\text{b} & (2,2) & (4,2) \\
\text{l} & (2,4) & (1,1) \\
\end{array}
\]

Our classification of $2 \times 2$ symmetric games does not apply to this one because $a = c$.

(a) Are there any strictly dominated or weakly dominated strategies?

(b) Find the pure strategy Nash equilibria.

(c) Check whether any pure strategy symmetric Nash equilibria that you found in part (b) correspond to evolutionarily stable states.

(d) Using the notation $\sigma = pb + ql$ (instead of $\sigma = p_1b + p_2l$), find the replicator equation for this game. Answer:

\[
\begin{align*}
\dot{p} &= p(2p + 4q - (p(2p + 4q) + q(2p + q))) \\
\dot{q} &= q(2p + q - (p(2p + 4q) + q(2p + q))).
\end{align*}
\]

(e) Use $p + q = 1$ to reduce this system of two differential equations to one differential equation in the variable $p$ only. Answer:

\[
\dot{p} = 3p(1 - p)^2.
\]
(f) Sketch the phase portrait on the interval $0 \leq p \leq 1$, and describe in words what happens.

2. Consider the generalization of the rock–paper–scissors game given in Gintis, problem 12.13. In class we considered the case $\alpha = 0$. In this problem we’ll consider $\alpha \neq 0$.

   (a) Show that there are no pure strategy Nash equilibria.

   (b) There is one mixed strategy Nash equilibrium, $p_1 = p_2 = p_3 = \frac{1}{3}$.

   (You don’t have to check this.)

   (c) Find the replicator equation for this game. P. 367 will help you check your answer.

   (d) Use $p_3 = 1 - p_1 - p_2$ to reduce this system of three differential equations to two differential equation in the variables $p_1$ and $p_2$ only. Again p. 367 will help you check your answer.

   (e) In the region $p_1 \geq 0$, $p_2 \geq 0$, $p_1 + p_2 \leq 1$, the only equilibria are the corners and $(\frac{1}{3}, \frac{1}{3})$. The phase portrait on the boundary of the triangle is the same as what we found in class for the case $\alpha = 0$. To get some idea of the phase portrait in the interior of the triangle, calculate the eigenvalues of the linearization at $(\frac{1}{3}, \frac{1}{3})$. Again p. 367 will help you check your answer. You should find that they have negative real part if $\alpha < 0$ and positive real part if $\alpha > 0$. What does this mean?