

MA 440 Homework 8

S. Schechter

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1. Problem 6.9 in Gintis. The answer to part a is given below, but you should explain the numbers. In part c, ignore the last four questions.

		Iraq	
		2	4
Iran	2	(46,42)	(26,44)
	4	(52,22)	(32,24)

2. Problem 6.12 in Gintis. In this problem the i th student's strategy is a number x_i , $0 \leq x_i \leq 1$, which represents the amount that student chooses to deposit in the public account. A strategy profile is therefore a 10-tuple $(x_1, x_2, \dots, x_{10})$ with $0 \leq x_i \leq 1$ for each i .

Ignore questions a–d, and instead answer the following.

- (a) Find the i th player's payoff function $\Pi_i(x_1, x_2, \dots, x_{10})$. (The answer is $\Pi_i(x_1, x_2, \dots, x_{10}) = 1 - x_i + \frac{1}{2}(x_1 + x_2 + \dots + x_{10})$; you should explain this.)
- (b) Show that each player has a strictly dominant strategy: Whatever the other players do, contribute nothing. (For example, consider player 1. Given any choices (x_2, \dots, x_{10}) by the other players, player 1 maximizes his payoff by choosing $x_1 = 0$.) Therefore the only Nash equilibrium is $(0, 0, \dots, 0)$, at which each player's payoff is 1.
- (c) Suppose the game is repeated every day. Consider the following strategy σ_x , where $0 < x \leq 1$: "I will contribute x dollars on day 0. If every other player contributes at least x dollars on day k , I

will contribute x dollars on day $k + 1$. If any player contributes less than x dollars on day k , I will contribute nothing on every subsequent day.” Show that if $\delta \geq \frac{1}{9}$, then it is a Nash equilibrium for every player to use the strategy σ_x with the same x . (In other words, $(\sigma_x, \sigma_x, \dots, \sigma_x)$ is a Nash equilibrium.)

3. Problem 4.23. Do the following subproblems instead of those suggested in the text.

- (a) Each firm has two strategies, M and E . Give the payoffs to each triple of pure strategies. You should organize your answer by giving two 2×2 matrices; see section 3.9 of the online notes.
- (b) Show that every triple of pure strategies except (M, M, M) and (E, E, E) is a Nash equilibrium.
- (c) Suppose firm 1 uses the mixed strategy $xM + (1 - x)E$, firm 2 uses the mixed strategy $yM + (1 - y)E$, and firm 3 uses the mixed strategy $zM + (1 - z)E$. Show that the payoff functions are

$$\begin{aligned}\Pi_1(x, y, z) &= 2(1 - x)yz + x(1 - y)(1 - z), \\ \Pi_2(x, y, z) &= 2(1 - y)xz + y(1 - x)(1 - z), \\ \Pi_3(x, y, z) &= 2(1 - z)xy + z(1 - x)(1 - y).\end{aligned}$$

- (d) Suppose one player uses the pure strategy M and one uses the pure strategy E . Show that any mix of strategies by the third player yields a Nash equilibrium. (For example, for any z with $0 \leq z \leq 1$, $(M, E, zM + (1 - z)E)$ is a Nash equilibrium.)
- (e) Show that there is no Nash equilibrium in which exactly one firm uses a pure strategy. Suggestion: Suppose firm 3 uses the pure strategy M , so that $z = 1$. If there is a Nash equilibrium with $0 < x < 1$ and $0 < y < 1$, then we must have

$$\frac{\partial \Pi_1}{\partial x}(x, y, 1) = 0 \text{ and } \frac{\partial \Pi_2}{\partial y}(x, y, 1) = 0.$$

- (f) Find a Nash equilibrium in which no firm uses a pure strategy.

4. This problem is based on a scene in the movie “A Beautiful Mind.” n men walk into a bar. In the bar is one extremely attractive woman and many attractive women. Each man has two possible pure strategies:

- S: Approach one of the attractive women. (The safe strategy.)
- R: Approach the extremely attractive woman. (The risky strategy.)

The payoffs are:

- $a > 0$ to each man who uses strategy S . (There are many attractive women in the bar; the strategy of approaching one of them will succeed.)
 - If there is a unique man who uses strategy R , his payoff is $b > a$. If more than one man uses strategy R , they all have payoff 0. (The extremely attractive woman doesn't enjoy being pestered and leaves.)
- (a) Find all pure strategy Nash equilibria of this n -person game.
 - (b) Find a mixed strategy Nash equilibrium in which all n men use the same mixed strategy $pS + (1 - p)R$.
 - (c) In the Nash equilibrium of part (b), for large n , what is the approximate probability that at least one man approaches the extremely attractive woman?