MA 440 Homework 6 Revised

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1. Gintis, problem 3.22. Don't turn in. Answers are in back of text.

2. At the end of season 1 of the television show Survivor, there were three contestants left on the island: Rudy, Kelly, and Rich. They were engaged in an "immunity challenge," in this case a stamina contest. Each contestant had to stand on an awkward support with one hand on a central pole. If the contestant's hand lost contact with the pole, even for an instant, the contestant was out. Once two contestants were out, the third contestant was the winner of the immunity challenge.

   The winner of the immunity challenge would then choose one of the other two contestants to kick off the island.

   Once there were only two contestants remaining on the island, a jury consisting of seven contestants who had recently been voted off the island would decide which of the two was the winner. The winner would get $1 million.

   We pick up the story when the immunity contest has been going for 1 1/2 hours. Rudy, Kelly, and Rich are still touching the pole.

   Rich has been thinking about the following considerations (as he later explained to the camera):

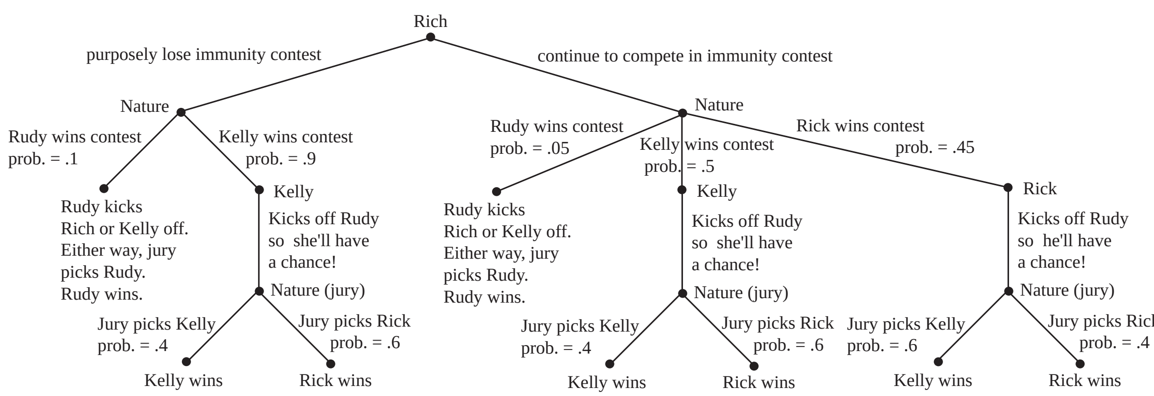
   - Rich and Kelly are strong young people. Rudy is much older. In addition, Kelly has become known for her stamina. Rich estimates that the probability of each winning the contest is Rich .45, Kelly .50, and Rudy .05. Rich further estimates that if he is the first contestant to lose touch with the pole, Kelly's probability of winning would be .9, and Rudy's would be .1.

- Rudy is much more popular with the jury than either Rich or Kelly. Rich figures that if Rudy is one of the last two contestants on the island, the jury is certain to pick Rudy as the winner.
- Rich and Kelly are equally popular with the jurors. However, if Rich or Kelly wins the immunity contest and kicks the popular Rudy off the island, some jurors might be made unhappy. Rich estimates that if he and Kelly are the last contestants on the island, but he has kicked off Rudy, there is a .4 chance the jury would pick him and a .6 chance it would pick Kelly. On the other hand, if he and Kelly are the last contestants on the island, and Kelly has kicked off Rudy, there is a .6 chance the jury would pick him and a .4 chance it would pick Kelly.

Rich is thinking about stepping away from the pole, thereby losing the immunity contest on purpose. Should he do it?

Make sure I can follow your reasoning.

The following diagram illustrates the situation.



(If you want to know what actually happened, rent the video!)

3. Boss Gorilla is boss of a Gorilla Group. Other gorillas are out there who might challenge him. When a visiting gorilla appears, it can do one of two things:

   - Challenge Boss Gorilla.
   - Leave.

If a visiting gorilla challenges Boss Gorilla, Boss Gorilla has two choices:

- Acquiesce. In this case the visiting gorilla joins Gorilla Group and becomes co-boss with Boss Gorilla.
- Fight.

There are two types of visiting gorillas: Tough and Weak. The Tough ones will win a fight with Boss Gorilla. The Weak ones will lose.

The probability that a visiting gorilla is Tough is p. The probability that he is Weak is $1 - p$. All gorillas know these probabilities. In addition, a visiting Gorilla knows which type he is, but Boss Gorilla does not.

We will view this as a game with three players: Tough Visiting Gorilla (T), Weak Visiting Gorilla (W), and Boss Gorilla (B).

Tough Visiting Gorilla has two strategies: Challenge (C) or Leave (L). Weak Visiting Gorilla has the same two strategies. Boss Gorilla has two strategies: Fight if Challenged (F) or Acquiesce if Challenged (A).

The game tree on the next page illustrates the situation and gives the payoffs.

You may assume that $0 < p < 1$.

(a) Explain why Tough Visiting Gorilla's strategy C strictly dominates his strategy L. (In other words, no matter what p is and no matter what the other gorillas do, Tough Visiting Gorilla will get a better payoff from C than from L.) Therefore *we will assume Tough Visiting Gorilla uses* C, and just consider the remaining two gorillas.

(b) Write down the 2×2 matrix that gives the payoffs to Weak Visiting Gorilla and Boss Gorilla from their strategy choices, assuming Tough Visiting Gorilla uses C. (Suggestion: When you write down Weak Visiting Gorilla's payoffs, you may be tempted to multiply them all by $1 - p$. Since you are just going to compare these numbers to each other, there is no need to do this. When you write down Boss Gorilla's payoffs, remember to take into account both types of visiting gorillas and their probabilities, since Boss Gorilla does not know who he is dealing with. If you do this correctly, three of Boss Gorilla's payoffs will have a p in them.)

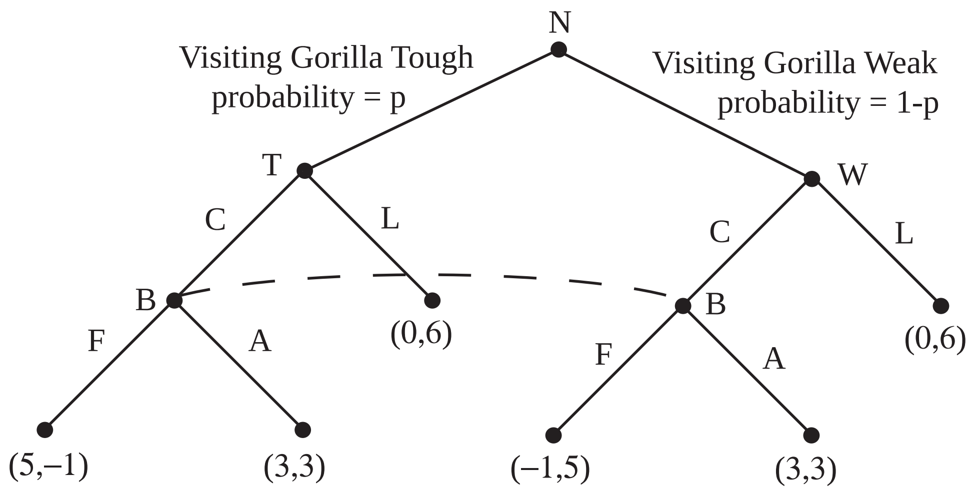


Figure 1: T is Tough Visiting Gorilla, W is Weak Visiting Gorilla, B is Boss gorilla. The visiting gorilla's payoffs are given first. If a visiting gorilla leaves, payoffs are 0 to him and 6 to Boss Gorilla. This is the value of being boss of Gorilla Group. Therefore, if a visiting gorilla challenges and Boss Gorilla acquiesces, payoffs are 3 to each. If the gorillas fight, payoffs are -1 to the loser (for injuries sustained) and 5 to the winner (6 for getting to be boss of Gorilla Group, minus 1 for injuries sustained). The dashed line indicates two nonterminal vertices in the same information set: if a visiting gorilla challenges, Boss Gorilla does not know if he is Tough or Weak.

(c) Find the Nash equilibria of this 2×2 payoff matrix. (Hint: there are two cases, $p > \frac{1}{3}$, and $p < \frac{1}{3}$. You can ignore $p = \frac{1}{3}$.)

(d) Explain why each of the Nash equilibria you just found, when combined with Tough Visiting Gorilla's strategy C, gives a Nash equilibrium of the 3-player game.

4. Gintis, problem 4.14. Assume $c_b \neq 2c_\ell$. In part (a), for each pair of pure strategies, find conditions under which that pair can be a Nash equilibrium, or state that that pair can never be a Nash equilibrium. (For example, you should find that (b, b) is a Nash equilibrium if $c_b > 2c_\ell$.) Do parts (b) and (c) as stated.