

MA 440 Homework 2

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1. In a certain town, there are two stores, a grocery store and a gas station. The grocery store charges p_1 dollars per pound for food, and the gas station charges p_2 dollars per gallon for gas. The grocery store sells q_1 pounds of food per week, and the gas station sells q_2 gallons of gas per week. The quantities q_1 and q_2 are related to the prices p_1 and p_2 as follows:

$$\begin{aligned}q_1 &= 10 - 2p_1 - p_2, \\q_2 &= 10 - p_1 - 2p_2.\end{aligned}$$

Thus, if the price of food or gas rises, less of *both* is sold.

Let r_1 be the revenue of the grocery store and r_2 the revenue of the gas station. Then

$$\begin{aligned}r_1 &= q_1 p_1 = (10 - 2p_1 - p_2)p_1 = 10p_1 - 2p_1^2 - p_1 p_2, \\r_2 &= q_2 p_2 = (10 - p_1 - 2p_2)p_2 = 10p_2 - p_1 p_2 - 2p_2^2.\end{aligned}$$

We interpret this as a game with two players, the grocery store (player 1) and the gas station (player 2). The payoff to each player is its revenue.

Suppose the grocery store chooses its price p_1 first, and then the gas station, knowing p_1 , chooses its price p_2 . If the grocery store uses backward induction to choose p_1 , what price will it choose? What will be the corresponding p_2 , and what will be the revenue of each store?

- (a) First do the problem allowing p_1 and p_2 to be any real numbers, even negative numbers, and even numbers that produce negative values of q_1 and q_2 .
- (b) Next redo the problem with the following additional stipulations
 - i. $0 \leq p_1 \leq 5$.

- ii. $0 \leq p_2 \leq 5$.
- iii. If $10 - 2p_1 - p_2 \leq 0$ then $q_1 = 0$. (If the prices are too high, no food is sold.)
- iv. $10 - p_1 - 2p_2 \leq 0$ then $q_2 = 0$. (If the prices are too high, no gas is sold.)