1. In a certain town, there are two stores, a grocery store and a gas station. The grocery store charges $p_1$ dollars per pound for food, and the gas station charges $p_2$ dollars per gallon for gas. The grocery store sells $q_1$ pounds of food per week, and the gas station sells $q_2$ gallons of gas per week. The quantities $q_1$ and $q_2$ are related to the prices $p_1$ and $p_2$ as follows:

\[
q_1 = 10 - 2p_1 - p_2, \quad q_2 = 10 - p_1 - 2p_2.
\]

Thus, if the price of food or gas rises, less of both is sold.

Let $r_1$ be the revenue of the grocery store and $r_2$ the revenue of the gas station. Then

\[
\begin{align*}
  r_1 &= q_1p_1 = (10 - 2p_1 - p_2)p_1 = 10p_1 - 2p_1^2 - p_1p_2, \\
  r_2 &= q_2p_2 = (10 - p_1 - 2p_2)p_2 = 10p_2 - p_1p_2 - 2p_2^2.
\end{align*}
\]

We interpret this as a game with two players, the grocery store (player 1) and the gas station (player 2). The payoff to each player is its revenue.

Suppose the grocery store chooses its price $p_1$ first, and then the gas station, knowing $p_1$, chooses its price $p_2$. If the grocery store uses backward induction to choose $p_1$, what price will it choose? What will be the corresponding $p_2$, and what will be the revenue of each store?

(a) First do the problem allowing $p_1$ and $p_2$ to be any real numbers, even negative numbers, and even numbers that produce negative values of $q_1$ and $q_2$.

(b) Next redo the problem with the following additional stipulations

i. $0 \leq p_1 \leq 5$. 

ii. \( 0 \leq p_2 \leq 5 \).

iii. If \( 10 - 2p_1 - p_2 \leq 0 \) then \( q_1 = 0 \). (If the prices are too high, no food is sold.)

iv. \( 10 - p_1 - 2p_2 \leq 0 \) then \( q_2 = 0 \). (If the prices are too high, no gas is sold.)