

MA 440 Homework 10

S. Schechter

November 21, 2008, revised December 1, 2008

1. Consider the game of lions and antelopes (4.14 in Gintis) in the case $c_b = 2c_l$. For definiteness we'll take $c_b = 4$ and $c_l = 2$. The payoff matrix is then

		lion 2	
		b	l
lion 1	b	(2,2)	(4,2)
	l	(2,4)	(1,1)

- (a) Are there any strictly dominated or weakly dominated strategies?
(b) Find the pure strategy Nash equilibria.
(c) Check whether any pure strategy symmetric Nash equilibria that you found in part (b) correspond to evolutionarily stable states.
(d) Using the notation $\sigma = pb + ql$ (instead of $\sigma = p_1b + p_2l$), find the replicator equation for this game. Answer:

$$\begin{aligned}\dot{p} &= p(2p + 4q - (p(2p + 4q) + q(2p + q))), \\ \dot{q} &= q(2p + q - (p(2p + 4q) + q(2p + q))).\end{aligned}$$

- (e) Use $p + q = 1$ to reduce this system of two differential equations to one differential equation in the variable p only. Answer:

$$\dot{p} = 3p(1 - p)^2.$$

- (f) Sketch the phase portrait on the interval $0 \leq p \leq 1$, and describe in words what happens.

2. Consider the generalization of the rock-paper-scissors game given in problem 9.13. In class we considered the case $r = 1$, $s = -1$. Let's instead assume just $r > 0$ and $s < 0$.

- (a) Show that there are no pure strategy Nash equilibria.
- (b) Show that there are no Nash equilibria in which each player uses the same two strategies with positive probability and the other strategy with 0 probability. (Just check the possibility that both players use strategies 1 and 2 with positive probability and strategy 3 with 0 probability. The other cases are the same.)
- (c) There is one mixed strategy Nash equilibrium, $\alpha = \beta = \gamma = \frac{1}{3}$. Is it evolutionarily stable? (The answer depends on $r + s$.)
- (d) Find the replicator equation for this game. Answer:

$$\begin{aligned}\dot{\alpha} &= (r\beta + s\gamma - (\alpha\beta + \beta\gamma + \gamma\alpha)(r + s))\alpha, \\ \dot{\beta} &= (r\gamma + s\alpha - (\alpha\beta + \beta\gamma + \gamma\alpha)(r + s))\beta, \\ \dot{\gamma} &= (r\alpha + s\beta - (\alpha\beta + \beta\gamma + \gamma\alpha)(r + s))\gamma.\end{aligned}$$

- (e) Use $\alpha + \beta + \gamma = 1$ to reduce this system of three differential equations to two differential equation in the variables α and β only. The answer is on p. 452.
- (f) In the region $\alpha \geq 0$, $\beta \geq 0$, $\alpha + \beta \leq 1$, the only equilibria are the corners and $(\frac{1}{3}, \frac{1}{3})$. The phase portrait on the boundary of the triangle is the same as what we found in class for the case $r = 1$, $s = -1$. To get some idea of the phase portrait in the interior of the triangle, calculate the eigenvalues of the linearization at $(\frac{1}{3}, \frac{1}{3})$. You should find that they have negative real part if $r + s > 0$ and positive real part if $r + s < 0$. What does this mean?