

MA 440 Homework 11

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December 1, 2007

1. Problem 9.17. The payoff matrix is in the text. Consider this as an asymmetric evolutionary game, and analyze it as we did Big Monkey and Little Monkey:
 - (a) Check for pure strategy Nash equilibria.
 - (b) Assume the buyer uses strategy I with probability p and strategy T with probability $1 - p$, and the seller uses strategy H with probability q and strategy D with probability $1 - q$. Derive the replicator equation (\dot{p} and \dot{q} equations only).
 - (c) Find all equilibria of the replicator equation in the region $0 \leq p \leq 1$, $0 \leq q \leq 1$.
 - (d) Find where \dot{p} and \dot{q} are 0, and use this information to sketch the vector field in the $0 \leq p \leq 1$, $0 \leq q \leq 1$. Be sure to show the direction of flow on the boundary of the region. One cannot tell from the vector field whether there are closed orbits.
 - (e) Find the eigenvalues of the linearization at the equilibrium $(\frac{1}{2}, \frac{1}{2})$. They don't help.
 - (f) Using separation of variables, show that the function $f(p, q) = (p - p^2)(q - q^2)$ is constant on solutions. Does this help you decide whether there are closed orbits?

The evolution of cooperative social behavior is a topic of current interest in biology. For a recent review article, see Nature, 27 October 2005 (<http://www.nature.com/nature/journal/v437/n7063/pdf/nature04131.pdf>).