This problem set is based on problem 2.11 in the text, but I’ve made it a little more precise.

A Ming vase is sold at auction. The auction works like this. The auctioneer calls out the price $k$ dollars. Any bidder who wants may raise her hand.

1. If more than one bidder raises her hand, the auctioneer calls out the price $k + 1$ dollars.
2. If exactly one bidder raises her hand, the auction is over, and the vase is sold to that bidder for $k$ dollars.
3. If no bidder raises her hand, the auction is over, but the vase is not sold to anyone.

The bidders raise their hands simultaneously. The auctioneer begins at 1 dollar.

There are $n$ bidders, $n \geq 2$. The value of the vase to bidder $i$ is $v_i$ dollars; $v_i$ is a positive integer. The payoff to bidder $i$ is 0 if bidder $i$ does not win the vase, and is $v_i$ minus the price if player $i$ does win the vase.

A strategy for bidder $i$ is simply the set of prices at which bidder $i$ will raise her hand, if the auctioneer calls out that price. For example, if bidder $i$ is willing to bid up to 5 dollars, her strategy is the set $\{1, 2, 3, 4, 5\}$. You may assume that each bidder’s strategy is a finite set. However, you should not assume that each bidder’s strategy is a set of consecutive integers that starts with 1. For example, a possible strategy is $\{2, 4, 5\}$.

1. Let $s_i$ be a strategy for bidder $i$ in which the highest bid is $k$ dollars, with $k > v_i$. Let $t_i$ be the strategy for bidder $i$ that is obtained from $s_i$, ...
by deleting the bid $k$. Show that $t_i$ weakly dominates $s_i$. (Suggestion: For any choice of strategies by the other players, if the auction is over before the bidding reaches $k$ dollars, $t_i$ and $s_i$ give the same result. What if the bidding reaches $k$ dollars?)

2. Explain why problem 1 implies that every strategy of bidder $i$ in which some bid is higher than $v_i$ dollars is weakly dominated by a strategy in which no bid is higher than $v_i$ dollars.

3. Let $s_i$ be a strategy for bidder $i$ in which the highest bid is $k$ dollars, with $k \leq v_i$. Suppose $s_i$ does not include all bids from 1 to $k$. Let $l$ be the lowest bid that is not included in $s_i$. Let $t_i$ be the strategy for bidder $i$ obtained from $s_i$ by including the bid $l$. Show that $t_i$ weakly dominates $s_i$. (Suggestion: Consider the following cases: (1) The auction is over before the bidding reaches $l$ dollars. (2) The auction reaches $l$ dollars, but no bidder other than the $i$th bids $l$ dollars. (3) The auction reaches $l$ dollars, and exactly one bidder other than the $i$th bids $l$ dollars. (4) The auction reaches $l$ dollars, and two or more bidders other than the $i$th bid $l$ dollars.)

4. Consider the following collection of strategies for bidder $i$: $\{1\}$, $\{1, 2\}$, $\{1, 2, 3\}$, . . . , $\{1, 2, ..., v_i\}$. Explain why problems 1–3 imply that every strategy for bidder $i$ that is not in this collection is weakly dominated by one of the strategies in the collection.

5. Show that bidder $i$’s strategies $\{1\}$, $\{1, 2\}$, $\{1, 2, 3\}$, . . . , $\{1, 2, ..., v_i - 2\}$ are all weakly dominated by her strategies $\{1, 2, ..., v_i - 1\}$ and $\{1, 2, ..., v_i\}$.

6. Does either of bidder $i$’s strategies $\{1, 2, ..., v_i - 1\}$ and $\{1, 2, ..., v_i\}$ weakly dominate the other?