MA 425-002 Homework

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In class we stated the algebraic properties of $\mathbb{R}$, which are on p. 23 of the text, and we defined $a - b$ and $\frac{a}{b}$. Then we proved

(a) If $a + b = 0$, then $b = -a$.

(b) For any real number a, $a \cdot 0 = 0 \cdot a = 0$.

In doing the following problems, you can use the algebraic properties of $\mathbb{R}$; you can use results (a) and (b) above; and when you get to problem n, you can use problems $1, \ldots, n-1$.

1. Prove: if $a + x = b$, then $x = b - a$. (Suggestion: assume $a + x = b$ and add $-a$ to both sides.)

2. Prove: if $a \cdot b = 1$, then $b = \frac{1}{a}$. (Suggestion: assume $a \cdot b = 1$. Use (b) to show that $a \neq 0$. Then multiply both sides by $\frac{1}{a}$.)

3. Prove: if $a \neq 0$ and $a \cdot x = b$, then $x = \frac{b}{a}$.

4. Prove: if $a \neq 0$ and $b \neq 0$, then $\frac{1}{ab} = \frac{1}{a} \cdot \frac{1}{b}$. (Suggestion: Show that $ab \cdot (\frac{1}{a} \cdot \frac{1}{b}) = 1$. Then use (2).)

5. Prove: $-(-b) = b$. (Suggestion: we know that $(-b) + b = 0$. Use result (a).)

6. Prove: $(-1) \cdot a = -a$. (Suggestion: show that $a + (-1) \cdot a = 0$. Then use result (a).)

7. Prove: if $a \cdot b = 0$ then $a = 0$ or $b = 0$. (Here is a suggestion for a proof by contradiction. Assume $a \cdot b = 0$ and it is not true that $a = 0$ or $b = 0$. Then $a \cdot b = 0$, $a \neq 0$, and $b \neq 0$. Derive a contradiction by showing that the first two of these statements imply that $b = 0$.)