1. Group 1.

Do four problem from Group 1. Give careful proofs from first principles.

(a) Let $S$ be a nonempty subset of $\mathbb{R}$ that is bounded above. Let $u = \sup S$. Let $a > 0$. Let $aS = \{as : s \in S\}$. Show that $\sup aS = au$.

(b) Let $x_n = 2 + \frac{(-1)^n}{n^2}$. Prove that $x_n \to 2$.

(c) Let $(x_n)$ be a sequence such that $x_n \to x$. Suppose $x > a$. Prove that there is a number $N$ such that $x_n > a$ for all $n > N$.

(d) Let $(x_n)$ be a sequence such that (1) $x_n > a$ for every $n$ and (2) $x_n \to x$. Prove that $x \geq a$.

(e) Prove: If $(x_n)$ is a bounded increasing sequence and $u = \sup \{x_n : n \in \mathbb{N}\}$, then $x_n \to u$.

2. Group 2.

Answer all questions in Group 2.

(a) Let $(x_n)$ be a sequence such that (1) $x_n > a$ for every $n$ and (2) $x_n \to x$. Is $x > a$?

   i. Yes.

   ii. No.

   iii. Maybe.
(b) Let \((x_n)\) be a monotone increasing sequence. Does \((x_n)\) converge?
   i. Yes.
   ii. No.
   iii. Maybe.

(c) Let \((x_n)\) be a bounded sequence. Does \((x_n)\) converge?
   i. Yes.
   ii. No.
   iii. Maybe.

(d) Suppose \((x_n)\) converges and \((x_{n_k})\) is a subsequence of \((x_n)\). Does \((x_{n_k})\) converge?
   i. Yes.
   ii. No.
   iii. Maybe.

(e) Let \((x_n)\) be a sequence such that (1) \(1 < x_n < 2\) for every \(n\) and
   (2) \(x_{n+1} < x_n\) for every \(n\). Does \(x_n \to 1\)?
   i. Yes.
   ii. No.
   iii. Maybe.