MA 425 Test 2 Practice Problems

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On problems 1–5, give ϵ-δ-type proofs or use the squeeze theorem.

1. Suppose (x_n) is a Cauchy sequence. Without using the fact that a Cauchy sequence converges, prove that (x_n) is bounded.

2. Let
$$f(x) = \begin{cases} 0 & \text{if } x < 0, \\ x^2 & \text{if } x > 0 \text{ and } x \text{ is rational}, \\ -x^2 & \text{if } x > 0 \text{ and } x \text{ is irrational}. \end{cases}$$
Prove that $\lim_{x \to 0} f(x) = 0$.

3. Let $f : (a, b) \to \mathbb{R}$ and $g : (a, b) \to \mathbb{R}$ be functions. Suppose $\lim_{x \to a} f(x) = \infty$ and $\lim_{x \to a} g(x) = L$, where L is a finite number. Prove that $\lim_{x \to a} f(x) + g(x) = \infty$.

4. Let $f : A \to \mathbb{R}$ be a function and let $c \in A$. Assume that f is continuous at $x = c$. Suppose that $f(c) = d$ and $d > 0$. Prove that there is a number $\delta > 0$ such that if $x \in A$ and $|x - c| < \delta$ then $f(x) > \frac{2}{3}d$.

5. Let $f : A \to \mathbb{R}$ and $g : A \to \mathbb{R}$ be uniformly continuous functions. Prove that $f + g$ is uniformly continuous.

6. Suppose (x_n) converges and (x_{n_k}) is a subsequence of (x_n). Does (x_{n_k}) converge?

   (a) Yes.

   (b) No.

   (c) Maybe.

7. Let (x_n) be a bounded sequence. Does $\lim(x_n)$ exist?

(a) Yes.
(b) No.
(c) Maybe.

8. Let (x_n) be a bounded sequence. Does some subsequence of (x_n) have a limit?

   (a) Yes.
   (b) No.
   (c) Maybe.

9. Let (x_n) be a bounded sequence. Let (x_{n_k}) be a subsequence of (x_n) that is monotone increasing. Does $\lim(x_{n_k})$ exist?

   (a) Yes.
   (b) No.
   (c) Maybe.

10. Let f be a continuous function on the interval $(0, 1)$ such that $f(x) > 0$ for all x in $(0, 1)$. Is it possible that

$$\inf_{x \in (0,1)} f(x) = 0?$$

   (a) Yes.
   (b) No.

11. Let f be a continuous function on the interval $[0, 1]$ such that $f(x) > 0$ for all x in $[0, 1]$. Is it possible that

$$\inf_{x \in (0,1)} f(x) = 0?$$

   (a) Yes.
   (b) No.