MA 425-002 Homework

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1. Let (x_n) be a sequence and let x be a number. Suppose that for each $k \in \mathbb{N}$, there is a corresponding number N such that if $n > N$ then $|x_n - x| < \frac{1}{k}$. Prove that $x_n \to x$.

2. Sec. 3.4 problem 14. Suggestion: You are supposed to show that "there is" a subsequence of (x_n) that converges to s. The usual way to show that something exists is to exhibit it. In this problem, that means that you should explain how to construct a subsequence (x_{n_k}) of (x_n) that approaches s.

   Here is an idea of how to construct such a subsequence, with justifications left for you to fill in. In particular, when I say to choose something with a certain property, you need to explain why this can be done.

   (a) Choose n_1 such that $s - 1 < x_{n_1} < s$.

   (b) Suppose we have chosen $n_1 < n_2 < \ldots < n_k$ such that

   $$s - \frac{1}{1} < x_{n_1} < s, \quad s - \frac{1}{2} < x_{n_2} < s, \quad \ldots \quad s - \frac{1}{k} < x_{n_k} < s.$$

   Choose $n_{k+1} > n_k$ such that $s - \frac{1}{k+1} < x_{n_{k+1}} < s$.

   (c) The subsequence (x_{n_k}) converges to s

3. Sec. 3.5 problem 4.