MA 341-007 Test 1 Review Questions

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Warning: Not all topics are covered!

1. Translate into a differential equation: Sec. 1.1 problems 15 and 16.

2. Checking whether something is a solution of a differential equation: Sec. 1.2 problems 8 and 10.

3. Existence-uniqueness theorem: Sec. 1.2 problem 28. Also: What if the initial condition is $y(1) = 2$?

4. Direction fields: Sec. 1.3 problem 17. Use the method of isoclines to sketch the direction field in the region $x > 0$. Then try to sketch the solutions with $y(1) = 1$ and $y(1) = 5$. For these two solutions, as $x \to \infty$, what do you think $y$ approaches?

5. Euler’s method: Sec. 1.4 problem 6. Just do the points $x = 1.1, 1.2$. Don’t round.


7. Linear equations: Sec. 2.3 problems 16, 20.

8. Exact equations: Sec. 2.4 problem 12.

9. Mixing: Sec. 3.2 problem 2.

10. Mechanics: Sec. 3.4 problem 6. Initially the object is 100 m above the ground. First draw your coordinate axis!
Answers:

1. 16: \( \frac{dA}{dt} = kA^2 \).

2. Both are solutions. Just calculate \( \frac{du}{dx} \) and plug into the differential equation.

3. No, yes. Look at \( \frac{df}{dy} \) at the two points.

4. Suggestion: Look at the isoclines \( c = -2, -1, 0, 1, 2 \).

5. Approximation from Euler’s method: At \( x = 1.1, y = 0.1 \); at \( x = 1.2, y = 0.209 \).

6. \( y = \left(1 - \frac{1}{2} \ln(1 + x)\right)^2 \)

7. 16: \( y = \frac{1}{(x^2 + 1)^2} \left( \frac{x^5}{5} + \frac{x^4}{4} + x^2 - x + C \right) \)

20: \( y = \frac{3}{5} x^2 - \frac{1}{2} x + \frac{C}{x^3} \)

8. \( e^x \sin y - x^3 + y^{\frac{3}{2}} = C \)

9. \( x = 2.5 - 2.0e^{-12t}; t = \frac{25}{3} \ln 2 = 5.776 \).

10. If \( x \) increases as you go up, \( v = -4.9 + 24.9e^{-2t} \).

   If the initial position of the object is \( x = 0 \), \( x = -4.9t - 12.45e^{-2t} + 12.45 \).

   Then \( x = -100 \) when (after simplifying) \( 22.95 = t + 2.54e^{-2t} \).

   Solution according to Maple: \( t = 22.95 \). (Not surprising.)