\[
\int (f(u) + g(u)) \, du = \int f(u) \, du + \int g(u) \, du. \quad \int cf(u) \, du = c \int f(u) \, du.
\]
\[
\int u \, dv = uv - \int v \, du. \quad \int u^n \, du = \frac{u^{n+1}}{n+1}, \quad n \neq -1. \quad \int \frac{du}{u} = \ln |u|.
\]
\[
\int e^u \, du = e^u. \quad \int u^e \, du = (u-1)e^u. \quad \int e^{-u} \, du = -e^{-u} + C.
\]
\[
\int \frac{du}{u \ln u} = \ln |\ln u|.
\]
\[
\int a^u \, du = \frac{a^u}{\ln a}, \quad a > 0, \ a \neq 1. \quad \int \ln u \, du = \ln u \ln u - u. \quad \int \frac{du}{u^2 + a^2} = \frac{1}{a} \tan^{-1} \left( \frac{u}{a} \right)
\]
\[
\int \frac{du}{u^2 - a^2} = \frac{1}{2a} \ln \left[ \frac{u-a}{u+a} \right]. \quad \int \frac{du}{\sqrt{u^2 - a^2}} = \frac{1}{a} \sin^{-1} \left( \frac{u}{a} \right), \quad a^2 \geq u^2.
\]
\[
\int \frac{du}{\sqrt{u^2 - a^2}} = \frac{1}{a} \sin^{-1} \left( \frac{u}{a} \right), \quad a^2 \geq u^2. \quad \int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \ln \left[ \frac{a + \sqrt{a^2 - u^2}}{u} \right], \quad u > a > 0.
\]
\[
\int \sin u \, du = -\cos u. \quad \int \cos u \, du = \sin u. \quad \int \tan u \, du = \ln |\cos u|.
\]
\[
\int \cot u \, du = \ln |\sin u|. \quad \int \sec u \, du = \ln |\sec u + \tan u|. \quad \int \csc u \, du = -\ln |\csc u + \cot u|.
\]
\[
\int \sec^2 u \, du = \tan u. \quad \int \csc^2 u \, du = -\cot u. \quad \int \sec u \tan u \, du = \sec u.
\]
\[
\int \sin^2 u \, du = \frac{1}{2} u - \frac{1}{4} \sin 2u. \quad \int \cos^2 u \, du = \frac{1}{2} u + \frac{1}{4} \sin 2u. \quad \int \tan^2 u \, du = \tan u - u.
\]
\[
\int \cos^2 u \, du = \frac{\sin^2 u \cos u}{n} + \frac{n-1}{n} \int \sin^2 u \, du. \quad \int \cos^2 u \, du = \frac{\cos^2 u \sin u}{n} + \frac{n-1}{n} \int \cos^2 u \, du.
\]

*Note: An arbitrary constant is to be added to each indefinite integral.*
\[ \int u \sin u \, du = \sin u - u \cos u \quad \int u^n \sin u \, du = -u^n \cos u + n \int u^{n-1} \cos u \, du. \]

\[ \int u \cos u \, du = \cos u + u \sin u \quad \int u^n \cos u \, du = u^n \sin u - n \int u^{n-1} \sin u \, du. \]

\[ \int e^{au} \sin nu \, du = \frac{e^{au}(a \sin nu - n \cos nu)}{a^2 + n^2}, \quad \int e^{au} \cos nu \, du = \frac{e^{au}(a \cos nu + n \sin nu)}{a^2 + n^2}. \]

\[ \int \sin (a + b)u \, du = \frac{\sin(a + b)u}{2(a + b)} + \frac{\sin(a - b)u}{2(a - b)}, \quad a^2 \neq b^2. \]

\[ \int \cos (a + b)u \, du = \frac{\cos(a + b)u}{2(a + b)} + \frac{\cos(a - b)u}{2(a - b)}, \quad a^2 \neq b^2. \]

\[ \int \sin (a + b)u \, du = \frac{-\cos(a + b)u}{2(a + b)} + \frac{-\cos(a - b)u}{2(a - b)} , \quad a^2 \neq b^2. \]

\[ \int \sinh u \, du = \cosh u, \quad \int \cosh u \, du = \sinh u. \]

\[ \Gamma(t) = \int_0^\infty e^{-u^{t-1}} \, du, \quad t > 0; \quad \Gamma(\frac{1}{2}) = \sqrt{\pi}; \quad \text{and} \quad \Gamma(n + 1) = n!, \text{if } n \text{ is a positive integer.} \]

\[ f(x) = f(a) + f'(a)(x - a) + \frac{f''(a)}{2!}(x - a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x - a)^n + \cdots \quad \text{(Taylor series)} \]

\[ e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \quad \sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n + 1)!} \quad \cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} \]

\[ (1 - x)^{-1} = \sum_{n=0}^{\infty} x^n \quad (1 - x)^{-2} = \sum_{n=0}^{\infty} (n + 1)x^n \quad \ln(1 - x) = -\sum_{n=1}^{\infty} \frac{x^n}{n} \]

\[ \tan x = x + \frac{1}{3}x^3 + \frac{1}{3}x^5 + \frac{1}{3}x^7 + \frac{1}{3}x^9 + \cdots \quad \arctan x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n + 1} \]

\[ \arcsin x = x + \frac{1}{2}x^3 + \frac{1}{2}x^5 + \frac{1}{2}x^7 + \frac{1}{2}x^9 + \cdots \quad \arcsin x = \sum_{n=0}^{\infty} \frac{(-1)^{n+1} x^{2n+1}}{(2n + 1)!} \]

\[ J_0(x) = \frac{\sum_{k=0}^{\infty} (-1)^k x^{2k}}{(2k)!} \quad J_1(x) = \frac{\sum_{k=0}^{\infty} (-1)^k x^{2k+1}}{(2k+1)!} \quad J_2(x) = \frac{\sum_{k=0}^{\infty} (-1)^k x^{2k+2}}{k! \Gamma(n + k + 1)} \]

*Note: An arbitrary constant is to be added to each indefinite integral.*
LINEAR FIRST-ORDER EQUATIONS

A general solution to the first-order linear equation \( dy/dx + P(x)y = Q(x) \) is

\[
y(x) = \left[ \mu(x) \right]^{-1} \left( \int \mu(x)Q(x)dx + C \right), \quad \text{where} \quad \mu(x) = \exp \left( \int P(x)dx \right)
\]

METHOD OF UNDETERMINED COEFFICIENTS

To find a particular solution to the constant-coefficient differential equation

\[
a y^{(n)} + b y' + c y = P_m(t)e^{\alpha t},
\]

where \( P_m(t) \) is a polynomial of degree \( m \), use the form

\[
y_p(t) = t^r(A_m t^m + \cdots + A_1 t + A_0)e^{\alpha t};
\]

if \( r \) is not a root of the associated auxiliary equation, take \( s = 0 \); if \( r \) is a simple root of the associated auxiliary equation, take \( s = 1 \); and if \( r \) is a double root of the associated auxiliary equation, take \( s = 2 \).

To find a particular solution to the differential equation

\[
a y^{(n)} + b y' + c y = P_m(t)e^{\alpha t} \cos \beta t + Q_n(t)e^{\alpha t} \sin \beta t,
\]

where \( P_m(t) \) is a polynomial of degree \( m \) and \( Q_n(t) \) is a polynomial of degree \( n \), use the form

\[
y_p(t) = t^r(A_m t^m + \cdots + A_1 t + A_0)e^{\alpha t} \cos \beta t \\
+ t^r(B_m t^m + \cdots + B_1 t + B_0)e^{\alpha t} \sin \beta t,
\]

where \( k \) is the larger of \( m \) and \( n \). If \( \alpha + i\beta \) is not a root of the associated auxiliary equation, take \( s = 0 \); if \( \alpha + i\beta \) is a root of the associated auxiliary equation, take \( s = 1 \).

VARIATION OF PARAMETERS FORMULA

If \( y_1 \) and \( y_2 \) are two linearly independent solutions to \( ay'' + by' + cy = 0 \), then a particular solution to \( ay'' + by' + cy = g \) is \( y = v_1 y_1 + v_2 y_2 \), where

\[
v_1(t) = \int \frac{-g(t)y_2(t)}{aw[y_1, y_2](t)} \, dt, \quad v_2(t) = \int \frac{g(t)y_1(t)}{aw[y_1, y_2](t)} \, dt,
\]

and \( w[y_1, y_2](t) = y_1(t)y_2'(t) - y_1'(t)y_2(t) \).
<table>
<thead>
<tr>
<th>$f(t)$</th>
<th>$F(s) = \mathcal{L}{f}(s)$</th>
<th>$f(t)$</th>
<th>$F(s) = \mathcal{L}{f}(s)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(at)$</td>
<td>$\frac{1}{a} F\left(\frac{s}{a}\right)$</td>
<td>$\sqrt{\frac{s}{t}}$</td>
<td>$\sqrt{\frac{s}{\pi}}$</td>
</tr>
<tr>
<td>$e^{-at}f(t)$</td>
<td>$F(s-a)$</td>
<td>$\sqrt{\frac{s}{t}}$</td>
<td>$\frac{1}{2s} \sqrt{\frac{s}{\pi}}$</td>
</tr>
<tr>
<td>$f'(t)$</td>
<td>$sF(s) - f(0)$</td>
<td>$t^{-\frac{1}{2}}, \quad n = 1, 2, \ldots$</td>
<td>$\frac{\Gamma(r+1)}{s^r}$</td>
</tr>
<tr>
<td>$\cdots$</td>
<td>$\frac{1}{2s^{3/2}}$</td>
<td>$b$</td>
<td>$\frac{s-1}{s^2 + b^2}$</td>
</tr>
<tr>
<td>$t^r f(t)$</td>
<td>$\int_0^\infty F(u) , du$</td>
<td>$e^a \sin bt$</td>
<td>$\frac{b}{(s-a)^3 + b^2}$</td>
</tr>
<tr>
<td>$\int_0^t f(t) , dt$</td>
<td>$\frac{F(s)}{s}$</td>
<td>$e^a \cos bt$</td>
<td>$\frac{b}{(s-a)^2 + b^2}$</td>
</tr>
<tr>
<td>$(f \ast g)(t)$</td>
<td>$F(s) G(s)$</td>
<td>$\sin bt$</td>
<td>$\frac{b}{(s-a)^2 + b^2}$</td>
</tr>
<tr>
<td>$f(t + T) = f(t)$</td>
<td>$\frac{1}{1-e^{-\gamma T}} \int_0^\gamma f(t) , dt$</td>
<td>$\tanh bt$</td>
<td>$\frac{s}{s^2 - b^2}$</td>
</tr>
<tr>
<td>$(t-a) u(t-a), \quad a \geq 0$</td>
<td>$e^{-as} F(s)$</td>
<td>$2b^3 \quad (s^2 + 4b^2)^2$</td>
<td>$2b^3 \quad (s^2 + 4b^2)^2$</td>
</tr>
<tr>
<td>$g(t) u(t-a), \quad a \geq 0$</td>
<td>$e^{-as} \mathcal{L}{g(t+a)}(s)$</td>
<td>$2b^2 \quad (s^2 + b^2)^2$</td>
<td>$2b^2 \quad (s^2 + b^2)^2$</td>
</tr>
<tr>
<td>$\delta(t-a), \quad a \geq 0$</td>
<td>$e^{-as}$</td>
<td>$s^3 - b^3 \quad (s^2 + b^2)^3$</td>
<td>$s^3 - b^3 \quad (s^2 + b^2)^3$</td>
</tr>
<tr>
<td>$e^{at}$</td>
<td>$\frac{1}{s-a}$</td>
<td>$\sin bt \cos bt - \cos bt \sinh bt \quad 4b^3 \quad s^3 + 4b^2$</td>
<td>$\sinh bt \cos bt - \sin bt \quad 2b^2 s \quad s^2 - b^2$</td>
</tr>
<tr>
<td>$t^n, \quad n = 1, 2, \ldots$</td>
<td>$\frac{n!}{s^{n+1}}$</td>
<td>$\sinh bt \quad 2b^2 s \quad s^2 - b^2$</td>
<td>$\cos bt - \cos bt \quad \frac{2b^2 s}{s^2 - b^2}$</td>
</tr>
<tr>
<td>$e^{at} - e^{bt}$</td>
<td>$\frac{(a-b)s}{(s-a)(s-b)}$</td>
<td>$\cos bt - \cos bt \quad \frac{2b^2 s}{s^2 - b^2}$</td>
<td>$\cos bt - \cos bt \quad \frac{2b^2 s}{s^2 - b^2}$</td>
</tr>
<tr>
<td>$ae^{at} - be^{bt}$</td>
<td>$\frac{(a-b) s}{(s-a)(s-b)}$</td>
<td>$J_v(bt), v &gt; -1$</td>
<td>$\frac{(\sqrt{s^2 + b^2} - s)^v}{b^v \sqrt{s^2 + b^2}}$</td>
</tr>
</tbody>
</table>