MA 341-001 Some Review Questions on Secs. 9.1–9.6

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1. Consider the differential equation

$$\mathbf{x}'(t) = \begin{bmatrix} 0 & 1 \\ -4 & 0 \end{bmatrix} \mathbf{x}(t).$$

Let

$$\mathbf{x}_1(t) = \begin{bmatrix} \cos 2t \\ -2\sin 2t \end{bmatrix} \quad \text{and} \quad \mathbf{x}_2(t) = \begin{bmatrix} \sin 2t \\ 2\cos 2t \end{bmatrix}.$$

   (a) Check that $\mathbf{x}_1(t)$ is a solution. (Just substitute and check that it works.)

   (b) $\mathbf{x}_2(t)$ is also a solution. (You don't need to check this.) Show that $\mathbf{x}_1(t)$ and $\mathbf{x}_2(t)$ are linearly independent.

   (c) Give the general solution.

   (d) Give the solution that satisfies the initial condition

$$\mathbf{x}(0) = \begin{bmatrix} 3 \\ -3 \end{bmatrix}.$$

   (e) Check that one solution of

$$\mathbf{x}'(t) = \begin{bmatrix} 0 & 1 \\ -4 & 0 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 5e^{-t} \\ 0 \end{bmatrix}$$

   is

$$\mathbf{x}(t) = \begin{bmatrix} -e^{-t} \\ -4e^{-t} \end{bmatrix}.$$

   (f) Give the general solution.

2. Find the eigenvalues of the matrix

$$\mathbf{A} = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 3 & 5 \\ 2 & -2 & 1 \end{bmatrix}.$$

3. The matrix

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

has the eigenvalues $r = -1,\ 1,\ 2$. An eigenvector for the eigenvalue -1 is

$$\begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}.$$

(a) Find an eigenvector for the eigenvalue 1.

(b) Find an eigenvector for the eigenvalue 2.

(c) Give the general solution of $\mathbf{x}'(t) = \mathbf{A}\mathbf{x}(t)$.

4. The matrix

$$\mathbf{A} = \begin{bmatrix} 1 & 1 \\ -4 & 1 \end{bmatrix}$$

has the eigenvalues $r = 1 \pm 2i$. Find the general solution of $\mathbf{x}'(t) = \mathbf{A}\mathbf{x}(t)$.

Some answers:

1. (c) $\mathbf{x}(t) = c_1 \begin{pmatrix} \cos 2t \\ -2\sin 2t \end{pmatrix} + c_2 \begin{pmatrix} \sin 2t \\ 2\cos 2t \end{pmatrix}$

   (d) $\mathbf{x}(t) = \begin{pmatrix} 3\cos 2t - \frac{3}{2}\sin 2t \\ -6\sin 2t - 3\cos 2t \end{pmatrix}$

   (f) $\mathbf{x}(t) = c_1 \begin{pmatrix} \cos 2t \\ -2\sin 2t \end{pmatrix} + c_2 \begin{pmatrix} \sin 2t \\ 2\cos 2t \end{pmatrix} + \begin{pmatrix} -e^{-t} \\ -4e^{-t} \end{pmatrix}$

2. $2,\ 2 \pm 3i$.

3. (a) $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ (b) $\begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix}$

   (c)

$$\mathbf{x}(t) = c_1 e^{-t} \begin{pmatrix} -3 \\ 1 \\ 1 \end{pmatrix} + c_2 e^{t} \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} + c_3 e^{2t} \begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix}$$

4.

$$\mathbf{x}(t) = c_1 \begin{pmatrix} e^t \sin 2t \\ 2e^t \cos 2t \end{pmatrix} + c_2 \begin{pmatrix} -e^t \cos 2t \\ 2e^t \sin 2t \end{pmatrix}$$