MA 242-005 Final Examination

S. Schecter

May 8, 1998

This exam is in two parts. Please do not do Part I problems and Part II problems on the same paper. When you finish, fold the Part I questions together with your Part I answers, fold the Part II questions together with your Part II answers, put your name on each part, and turn in.

Part I

1. Find the equation of the plane that includes both the point (4, 2, 3) and the line
   \[ x = 2t, y = -1 + t, z = 3 - t. \]

2. Consider the space curve \( r(t) = < 2 \cos t, 3 \sin t, 4 >, 0 \leq t \leq \pi. \)
   
   (a) True or false: This curve lies in the plane \( z = 4. \)
   
   (b) True or false: This curve lies in the elliptic cylinder \( 9x^2 + 4y^2 = 36. \)
   
   (c) Make a sketch that shows the plane \( z = 4 \) and the curve \( r(t). \)
       Indicate the direction of increasing \( t. \)
   
   (d) Find the velocity vector \( v(t). \)
   
   (e) Find \( v(\pi/2), \) the velocity vector at time \( t = \pi/2, \) and add it to your sketch, with its tail at the appropriate point.
   
   (f) Find the speed \( v(t). \)
   
   (g) Find the acceleration vector \( a(t). \)
3. The value of the function \( f(x, y) = e^{2x} \cos 3y \) at \((x, y) = (0, 0)\) is easy to compute. Use partial derivatives to approximate the value of this function at \((x, y) = (-0.2, 0.1)\).

4. Let \( f(x, y, z) = e^{x^2y} + 2 \sin(y - z) \).

   (a) Calculate \( f_x, f_y, \) and \( f_z \).

   (b) Find the gradient of \( f \) at \((1, 0, \pi)\).

   (c) Find the directional derivative of \( f \) at \((1, 0, \pi)\) in the direction of the vector \(<2, -1, 1>\).

   (d) Find an equation for the tangent plane to the surface \( e^{x^2y} + 2 \sin(y - z) = 1 \) at \((1, 0, \pi)\).

5. Let \( f(x, y) = 6x^3 - 6xy + y^2 \). Find all critical points of \( f \), and determine if each is a local minimum, a local maximum, or a saddle point.
Part II

1. Evaluate

\[ \int \int \int _E \frac{e^{x^2+y^2}}{x^2+y^2} \, dV \]

where \( E \) is the solid region that lies between the cylinders \( x^2 + y^2 = 4 \) and \( x^2 + y^2 = 9 \), above the plane \( z = 0 \), and below the paraboloid \( z = x^2 + y^2 \).

2. Let \( E \) be the solid region that is inside the sphere \( x^2 + y^2 + z^2 = 25 \) and above the cone \( z = -\sqrt{x^2 + y^2} \). Assume the density is \( \rho(x, y, z) = z^2 \). Use a triple integral in spherical coordinates to find the mass of \( E \).

3. Evaluate the line integral \( \int _C y^2 \, dx + xz \, dz \), where \( C \) is the straight line from \((1, 0, 2)\) to \((0, 2, 5)\).

4. Let \( \mathbf{F}(x, y, z) = yz \mathbf{i} + (xz + 2yz) \mathbf{j} + (xy + y^2) \mathbf{k} \).

   (a) Check that \( \mathbf{F} \) is conservative by evaluating \( \text{curl} \ \mathbf{F} \).

   (b) Find a function \( f(x, y, z) \) such that \( \mathbf{F} = \nabla f \).

5. Let \( E \) be the solid region in the first octant that is bounded by the coordinate planes and the plane \( 2x + y + z = 6 \). Let \( S \) be the surface of \( E \), oriented outward. Let \( \mathbf{F}(x, y, z) = 2x \mathbf{i} + z \mathbf{j} + y \mathbf{k} \). Use the Divergence Theorem to find \( \int _S \mathbf{F} \cdot \mathbf{n} \, dS \).

6. Find the flux of \( \mathbf{F}(x, y, z) = 2x \mathbf{i} + 2y \mathbf{j} + 2z \mathbf{k} \) across the portion of the elliptic cylinder \( 9x^2 + 4y^2 = 36 \) that lies between the planes \( z = -1 \) and \( z = 1 \). The cylinder is oriented outward. Suggestion: the cylinder can be parameterized as follows:

\[ \mathbf{r}(\theta, z) = 2 \cos \theta \mathbf{i} + 3 \sin \theta \mathbf{j} + z \mathbf{k}, \ 0 \leq \theta \leq 2\pi, \ -1 \leq z \leq 1. \]

I have neither given nor received unauthorized aid on this test.

(Please sign.)