MA 114-001 Test 2

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Use your own paper to work the problems. Be sure to show your work.

When you finish, fold this paper lengthwise together with your work, so that this writing is on the outside. Write your name, row, and seat number above, and turn in.

1. For each of the following linear programming problems,

   (i) State whether the problem is standard or nonstandard.

   (ii) If the problem is nonstandard, rewrite it in the form we work with. For example, you may want to change a minimization problem to a maximization problem, and you may want to rewrite some of the inequalities.

   (iii) Once the problem is in the correct form, convert it to a system of equations, most of which contain slack variables. Don’t forget to include an equation for the objective function. All the equations should have the form: something = constant.

   (iv) Give the initial simplex tableau.

   Do not solve these problems. Just set them up.

(a) Maximize $P = 2x - 3y$

   subject to

   $-x + y \leq 1$

   $x + 2y \leq 4$

   $y \leq 1$

   $x \geq 0$

   $y \geq 0$
(b) Minimize \( C = 2x - 3y \)

subject to

\[
\begin{align*}
x - y &\leq 1 \\
2x + y &\leq 4 \\
x &\geq 0 \\
y &\geq 0
\end{align*}
\]

(c) Maximize \( P = 2x + 3y + z \)

subject to

\[
\begin{align*}
x - y + z &\geq 1 \\
2x + y - z &\geq 4 \\
x + 2y + 3z &\leq 12 \\
x &\geq 0 \\
y &\geq 0 \\
z &\geq 0
\end{align*}
\]

2. For the linear programming problem

Minimize \( C = -2x - y \)

subject to

\[
\begin{align*}
x + y &\leq 4 \\
x + 2y &\leq 6 \\
x &\geq 0 \\
y &\geq 0
\end{align*}
\]

the initial simplex tableau is

\[
\begin{pmatrix}
x & y & u & v & -C \\
1 & 1 & 1 & 0 & 0 & | & 4 \\
1 & 2 & 0 & 1 & 0 & | & 6 \\
-2 & -1 & 0 & 0 & 1 & | & 0
\end{pmatrix}
\]

(a) Graph the feasible region, labeling all vertices with their \((x, y)\) values, and labeling all boundary lines as \(x = 0\), \(u = 0\), etc.

(b) Solve the problem using the simplex method. This problem will go quickly if you use the pivot column recommended in the text.

(c) State the final basic solution, giving the values of \(x, y, u, v\) and \(P\).

(d) State the vertex on the graph that corresponds to this basic solution.
3. For the linear programming problem

Maximize $P = x - y$

subject to

\[
\begin{align*}
2x + 3y & \geq 18 \\
3x + 2y & \leq 18 \\
x & \geq 0 \\
y & \geq 0
\end{align*}
\]

the initial simplex tableau is

\[
\begin{array}{ccccc|c}
 x & y & u & v & P \\
\hline
-2 & -3 & 1 & 0 & 0 & -18 \\
3 & 2 & 0 & 1 & 0 & 18 \\
-1 & 1 & 0 & 0 & 1 & 0
\end{array}
\]

Solve this problem using the simplex method. (It should require two complete pivoting steps whichever way you do it.) Give the values of $x$ and $y$ at which $C$ is maximum, and give the maximum value of $C$. 