MA 114-001 S. Schecter

Test 1 Answers

① a) $x+2z=4$, $y+3z=1$ $\rightarrow$ $x=4-2z$, $y=1-3z$, z arbitrary

b) Last row says $0=1$; no solutions

② The system can be written

$$\begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}.$$

Multiplying on the left by the inverse, we get

$$\begin{pmatrix} -4 & 3 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -4 & 3 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} 1 \\ -2 \end{pmatrix}.$$

Multiplying out, we get

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -10 \\ 7 \end{pmatrix}$$

③

$$\left(\begin{array}{ccc|c} 0 & 2 & 6 & 4 \\ 1 & 0 & 4 & 2 \\ 3 & 2 & 10 & 2 \end{array}\right) \xrightarrow{R1 \leftrightarrow R2} \left(\begin{array}{ccc|c} 1 & 0 & 4 & 2 \\ 0 & 2 & 6 & 4 \\ 3 & 2 & 10 & 2 \end{array}\right) \xrightarrow{R3-3R1}$$

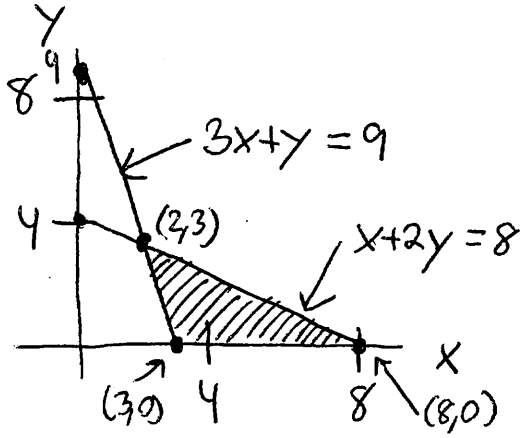
$$\left(\begin{array}{ccc|c} 1 & 0 & 4 & 2 \\ 0 & 2 & 6 & 4 \\ 0 & 2 & -2 & -4 \end{array}\right) \xrightarrow{\frac{1}{2}R2} \left(\begin{array}{ccc|c} 1 & 0 & 4 & 2 \\ 0 & 1 & 3 & 2 \\ 0 & 2 & -2 & -4 \end{array}\right) \xrightarrow{R3-2R2}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 4 & 2 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & -8 & -8 \end{array}\right) \xrightarrow{-\frac{1}{8}R3} \left(\begin{array}{ccc|c} 1 & 0 & 4 & 2 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right) \xrightarrow[R2-3R3]{R1-4R3}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

$x=-2$, $y=-1$, $z=1$

④



⑤

x	y	P = 4x+y
0	0	0
10	0	40
0	20	20
5	15	35

Maximum is $P = 40$ at $(x,y) = (10,0)$

⑥

x = number of rainbows

y = number of brookies

Maximize $W = 5x + 4y$

Subject to

$60x + 30y \leq 8{,}000$

$10x + 40y \leq 6{,}000$

$x \geq 0$

$y \geq 0$

Points ① a) 8 b) 4 ② 12 ③ 24 ④ 20 ⑤ 16 ⑥ 16