Heavy Traffic Limits in a Wireless Queueing Model with Long Range Dependence

Robert T. Buche, Arka Ghosh, Vladas Pipiras

***Abstract*— High-speed wireless networks carrying multimedia applications are becoming a reality and the transmitted data exhibit long range dependence and heavy-tailed properties. We consider the heavy traffic approach in working towards queue models under these properties, extending the model in [2]. Our focus is on the scalings used in the heavy traffic approach which are determined by combinations of the source rate of an infinite source Poisson model of the arrival process, the tail distribution of data transmitted by these sources, and the rate of variation of the random process (channel process) modeling the wireless medium. A fundamental inequality between the exponent in the power tail distribution of the data from the source and the parameter specifying the rate of channel variations is obtain. This inequality is important in both the "fast growth" and "slow growth" regimes for the arrival process and along with the source rate is used to define the possible cases for obtaining limit models for the queueing process. Across the cases, the possible limit models include reflected Brownian motion, reflected stable Lévy motion, or reflected fractional Brownian motion.**

I. Introduction

The usefulness of the heavy traffic method for *wireline* systems under traditional traffic (for example, Poisson arrival of packets and finite second moments on the packet size and service times) is well established, having a large literature (see [13], [6] and the references therein). Precise modeling of the physical queueing system may be intractable and the heavy traffic method scales time and the state space yielding a limit model giving the queue-size dynamics which retains the important features of the actual queueing system. In the traditional traffic cases, the limit model is given by reflected Brownian motion (RBM). Motivated by the prevalence of wireless systems, this heavy traffic model has been extended to the *wireless* case ([2]). The major difference from the wireline model is the random environment for transmission due to physical surroundings and mobility characteristics of the user. But like the wireline case, the limit queue models are given by RBM.

Recently, there has been a major paradigm shift for both wireline and wireless systems from traditional traffic models to long range dependent (LRD) and heavy-tailed (HT) stochastic models of data traffic. LRD stands for a very strong temporal dependence in a time series, and HT describes the tails of distribution functions giving unbounded second moments. This traffic has bursty behavior affecting network requirements (e.g. capacity and queue size) and is important to model. These effects have a significant influence on the network design and are being incorporated in planning for the next generation 4G wireless networks (e.g. [9]).

Following this paradigm shift, there has been some recent work on heavy traffic modeling of queueing systems under LRD/HT assumptions on the data traffic in wireline systems (see [3], [5], [4]). In these papers reflected fractional Brownian motion (RFBM) models for the queue process are obtained. The arrival and departure process models in these papers are such that the same scaling of time and state space are natural for both the arrival and departure process; in other words, the same scalings lead to nontrivial (i.e., nonzero or nondivergent) limit processes. In this paper, to the best of our knowledge, we give first results for extending the heavy traffic method under LRD/HT assumptions to a wireless queueing system, obtaining a set of limit models (see Fig. 1).

A major difference from the wireline case is that in the wireless models we consider there are different scalings yielding nontrivial limit models for the arrival and departure processes. In particular, we consider wireline arrivals but departures through a wireless medium. The arrival process models multimedia transmissions using an infinite source Poisson model: sources turn on according to a Poisson process with intensity (rate) λ, transmit at rate 1, and are on for a random interval which is heavy-tailed with parameter α (more detail is in Section II). The state space scaling for the arrivals depends on the growth condition of $\lambda = \lambda(T) \uparrow \infty$ with the scaling parameter $T \to \infty$: under a "slow growth" condition on $\lambda(T)$ (see (17) and (21)) one obtains a stable Lévy motion limit model and under a fast growth condition (see (16) and (20)) a fraction Brownian motion model. The departure process is through the wireless medium and the state space is scaled according to the rate of channel variations given by parameter ν (see (1)) and the weak dependence assumptions on the channel process lead to a Brownian motion limit model. Determining the scaling to use in the heavy traffic method for obtaining a queue model is more complex than the wireline case. More precisely, given the "operating conditions" or characteristics of the arrivals, $\lambda(T)$ and α, and those of the departures, ν, the question is: what is the "right" scaling to use to obtain nontrivial limit models, i.e., according to the arrivals or departures? This is tantamount to the question: given the operating conditions on the arrivals and departures, what is the appropriate limit model characterizing the queue content size: reflected Brownian motion (RBM), reflected fractional Brownian motion (RFBM), or reflected stable Lévy motion

Supported by the National Science Foundation grants DMS-0608669 (RB), DMS-0608634 (AG), DMS-0608663 (VP).

R. Buche is with the Mathematics Department, NC State Univ., Raleigh, NC 27695, rtbuche@unity.ncsu.edu

A. Ghosh is with the Statistics Department, Iowa State Univ., Ames, IA 50011, apghosh@iastate.edu

V. Pipiras is with the Statistics and Operations Research Department, UNC-Chapel Hill, NC, 27599, pipiras@email.unc.edu

(RSLM)? This question is answered in this paper: more generally, we give regions of operating conditions with their scalings and resulting limit model. What is surprising is that simple conditions emerge for defining these regions; furthermore, a simple inequality (see (23)) relating the channel rate (i.e., ν) and heavy-tails in the arrivals (i.e., α) is used in defining the regions. This is described in Section II. In Section III is a short summary.

In the remainder of this section we briefly highlight the heavy traffic method and results of [2]. Then in Section II we replace the arrival process with an infinite source Poisson model for multimedia traffic and derive the various queue limit models. A one-dimensional infinite queue with wireline arrivals and departures through a wireless medium is considered. We restrict our analysis to a one-dimensional case in order to focus on the development of the heavy traffic scalings, avoiding the complications of the reflected process in higher dimensions. In the 1-D case, the reflection direction is simply always in the positive queue direction. But in higher dimensions, there are complications surrounding the reflected process (unlike the wireline case) due to the possibility of multiple sets of reflection directions on the boundary of the queue state space, each set corresponding to a channel state (see [2] for some furthur discussion). Nonetheless, the *scaling* results here for the 1-D case are expected to carry over to higher dimensions.

Heavy traffic queue model: SRD/LT data assumptions. The following is a summary of the heavy traffic wireless queueing model adapted from [2] for a 1-D queue. In particular, an equivalent model is given where time is accelerated using the scaling parameter T instead of the rates being scaled.

In the heavy traffic method, one considers an embedded sequence of (scaled) queueing systems (Eq. (9) below) indexed by scaling parameter T. The mean arrival rate and departure rate of the data in the scaled systems are $O(T)$. The channel state is modeled by a Markov or semi-Markov process denoted process

$$L^T(t) = L(T^\nu t) \quad 0 < \nu < 1, \tag{1}$$

so that the rate of channel variations is slower than the rate of arrivals. This is reasonable as the channel coherence times (how long the channel is in a given state) are typically much longer than the service times for packets. We index the channel states, $L^T(t) = j$, $j \in \mathcal{J}$, $|\mathcal{J}|$ finite, where the index j corresponds to a transmission rate per unit power, $\bar{\lambda}^d(j)$. The state space scaling is according to the channel process and is given by $1/T^\gamma$, where $\gamma = 1 - \nu/2$. This "heavy traffic scaling" leads to a RBM limit model, outlined below.

The limit model is obtained under heavy traffic conditions for the wireless model given by:

$$\begin{aligned} \Lambda^a T \doteq \bar{\lambda}^a \bar{v}^b T &= \sum_{j\in\mathcal{J}} \bar{\lambda}^d(j)\bar{p}(j)\pi(j)T, \quad \text{and} \\ T^{1/2+\nu/2} u^T(j,x^T) &\to u(j,x) \quad \text{for } j \in \mathcal{J}. \end{aligned} \tag{2}$$

The first condition is a balance between mean arrival and departure rates where π is the stationary distribution of the channel, $\bar{\lambda}^a$ is the "canonical" mean batch arrival rate, $\bar{v}^b$ is the mean batch size, and $\bar{p}(j)$ is a "nominal" power in channel state j. The second condition specifies $u^T \to 0$ in such a way that the limit model has a drift rate component depending on *reserve power* $u(j,x)$, dependent on channel state j and queue state x.

Applying the heavy traffic scaling with parameter T to the queue balance equations, we obtain

$$x^T(t) = x^T(0) + A^T(t) - D^T(t), \tag{3}$$

where x is the (scaled) size of the queue, A^T is the cumulative arrival process, and D^T is the cumulative departure process. The arrival model is given by the following. The l-th interarrival time for the T-th system is given by $\Delta_l^{a,T}$. Let $S^{a,T}(t)$ be $1/T$ times the number of batches that have arrived by real time Tt, and v_l^b be the l-th batch size. Then,

$$A^T(t) = M^{a,T}(t) + \frac{\bar{v}^b}{\bar{\Delta}^a T^\gamma} \sum_{l=1}^{[T\,S^{a,T}(t)]} \Delta_l^{a,T}, \tag{4}$$

where

$$M^{a,T}(t) = \frac{1}{T^\gamma} \sum_{l=1}^{[T\,S^{a,T}(t)]} \left[v_l^b - \bar{v}^b\right] + \frac{\bar{v}^b}{T^\gamma} \sum_{l=1}^{[T\,S^{a,T}(t)]} \left[1 - \frac{\Delta_l^{a,T}}{\bar{\Delta}^a}\right]$$

is the scaled variations of the arrival about the mean process. Assume that the centered and scaled sequence of processes

$$\begin{aligned} w^{a,T}(t) &= \frac{1}{\sqrt{T}} \sum_{l=1}^{[Tt]} \left[1 - \frac{\Delta_l^{a,T}}{\bar{\Delta}^a}\right] \quad \text{and} \\ w^{b,T}(t) &= \frac{1}{\sqrt{T}} \sum_{l=1}^{[Tt]} \left[v_l^b - \bar{v}^b\right] \end{aligned} \tag{5}$$

for the interarrival times and batch sizes are tight. This is typically the case in the traditional wireline setting and is appropriate here since the data is arriving to the queues via wireline. Note that the state space scaling $\gamma > 1/2$ used here is "stronger" than the standard $1/\sqrt{T}$ scaling for the wireline case. This implies that $M^{a,n}(t) \Rightarrow 0$. The sum term in equation (4) is the time t in the scaled system plus an error term $e^T(t)$ that goes to 0 as $T \to \infty$.

The departure process is given by

$$\begin{aligned} D^T(t) &= \frac{1}{T^\gamma} \int_0^{Tt} \sum_{j\in\mathcal{J}} I_{\{L^T(s)=j\}} \bar{\lambda}^d(j) \\ &\quad \times \left[\bar{p}(j) + u^T(j,x^T(s))\right] I_{\{x^T(s)>0\}}\, ds, \end{aligned} \tag{6}$$

where $I_{\{\cdot\}}$ is the indicator function and $I_{\{x^T(s)>0\}}$ constrains the queues from being negative. Equation (6) can be rewritten as

$$\begin{aligned} D^T(t) &= M^{d,T}(t) - T^{\nu/2}\Lambda^a t - z^T(t) \\ &\quad + \frac{1}{T^\gamma} \int_0^{Tt} \sum_{j\in\mathcal{J}} I_{\{L^T(s)=j\}} \bar{\lambda}^d(j) u^T(j,x^T(s))\, ds, \end{aligned}$$

where (under a change of variables)

$$M^{d,T}(t) = \frac{1}{T^{(1+\nu)/2}} \int_0^{T^{1+\nu}t} \left[\sum_{j\in\mathcal{J}} I_{\{L(s)=j\}} - \pi(j) \right] \bar{\lambda}^d(j)\bar{p}(j)\, ds, \quad (7)$$

and $z^T(t)$ is the reflection process which represents the work that could have been done, had the queues not been empty, and is given by

$$z^T(t) = \frac{1}{T^\gamma} \int_0^{Tt} \left(\sum_{j\in\mathcal{J}} I_{\{L^T(s)=j\}} \bar{\lambda}^d(j) \times [\bar{p}(j) + u^T(j, x^T(s))] \right) I_{\{x^T(s)=0\}}\, ds. \quad (8)$$

Combining the expressions above and neglecting the error $e^T(t)$, the prelimit equation is then given by

$$x^T(t) = x^T(0) + M^{a,T}(t) - M^{d,T}(t) + z^T(t) - \frac{1}{T^\gamma} \int_0^{Tt} \sum_{j\in\mathcal{J}} I_{\{L^T(s)=j\}} \bar{\lambda}^d(j) u^T(j, x^T(s))\, ds. \quad (9)$$

The main result in [2] is that (9) weakly converges (in $D[0,\infty)$ under the Skorohod J_1-topology) to the following RBM

$$x(t) = x(0) + \int_0^t b(u, x(s))\, ds + w(t) + z(t), \quad (10)$$

where $w(\cdot)$ is Brownian motion, $z(\cdot)$ is the reflection process and the drift rate is given by

$$b(u, x) = -\sum_j \bar{\lambda}^d(j) u(j, x) \pi(j). \quad (11)$$

II. Scalings for Limit Models

In this section we replace the arrival process in (4) by an infinite source Poisson process modeling multimedia traffic. From [11] it is known that the *arrival process* can be scaled and shown to converge in some (weak) sense to fractional Brownian motion (FBM) or stable Lévy motion (SLM) under a "fast growth" or "slow growth" condition, repectively (see (16) and (17) along with (20) and (21)). We will show that there are several possible scalings for nontrivial *queue* limit models and show which scaling to use depends on the characteristics of the arrival and channel processes.

Before discussing the possible queue limit models, we consider the arrival process modeled by an infinite source Poisson process. Following [11], the "fast growth" and "slow growth" conditions are stated along with the scalings and arrival limit models.

A. Background: arrival and departure limit models.

Heavy traffic queue model: LRD/HT assumptions. The infinite source Poisson process modeling the arrivals is given by

$$N_T(s) = \sum_{k=-\infty}^{\infty} 1_{\{\Gamma_k \le s < \Gamma_k + V_k\}},$$

where $\{\Gamma_k\}_{k=0}^\infty$ are Poisson arrivals with intensity $\lambda(T) \uparrow \infty$ and $\{V_k\}_{k=0}^\infty$ are i.i.d. positive variables with heavy tail distribution

$$\bar{F}_V(v) = P(V > v) = v^{-\alpha}\mathcal{L}(v), \quad (12)$$

$1 < \alpha < 2$, and $\mathcal{L}(v)$ is a slowly varying function at $v = \infty$. The scaled cumulative arrival process is given by

$$A^T(t) = \frac{1}{\lambda(T)} \int_0^{Tt} N_T(s)\, ds. \quad (13)$$

The normalizing factor $\lambda(T)$ in the above ensures that the mean arrival rate is $O(T)$:

$$EA^T(t) = tT\mu_{\text{on}}, \quad \mu_{\text{on}} = EV.$$

The channel process is as in Section I: $L^T(s) = L(T^\nu s)$, $0 < \nu < 1$. The heavy traffic condition is given by (2) except the first condition now take the form

$$\mu_{\text{on}} T = \sum_j \bar{\lambda}(j)\bar{p}(j)\pi(j)T. \quad (14)$$

Denoting the state space scaling as $\beta(T)$, analogous to equation (9) in Section I, we have for the system equation:

$$\begin{aligned} x^T(t) &= x^T(0) + \beta(T)\left(A^T(t) - Tt\mu_{\text{on}}\right) \\ &\quad - \frac{\beta(T)}{T^\nu} \int_0^{T^{1+\nu}t} \sum_j \left(1_{\{L(s)=j\}} - \pi(j)\right) \bar{p}(j)\, ds \\ &\quad - \beta(T) \int_0^{Tt} \sum_j 1_{\{L^T(s)=j\}} \bar{\lambda}(j) u^T(x)\, ds \\ &\quad + \beta(T) \int_0^{Tt} \sum_j 1_{\{L^T(s)=j\}} \bar{\lambda}(j) p(j) 1_{\{x^T(s)=0\}}\, ds \\ &\doteq x^T(0) + M_a^T(t) - M_d^T(t) \\ &\quad - \beta(T) \int_0^{Tt} \sum_j 1_{\{L^T(s)=j\}} \bar{\lambda}(j) u^T(x(s))\, ds + z^T(t). \end{aligned} \quad (15)$$

Fast and slow growth conditions on arrivals. Some of the possible limit models given below use state space scalings $\beta(T)$ determined from the arrival process. In particular, the results of [11] are used, where the infinite source Poisson model of the arrival process is shown to converge to SLM or FBM, depending on whether the source rate $\lambda(T)$ for the arrival process is in the slow growth or fast growth regime. In particular, the regimes are given by

$$\text{fast growth:} \quad \lim_{T\to\infty} \frac{b(\lambda(T)T)}{T} = \infty, \quad (16)$$

$$\text{slow growth:} \quad \lim_{T\to\infty} \frac{b(\lambda(T)T)}{T} = 0 \quad (17)$$

where $b(\cdot)$ is given by

$$b(v) = \left(\frac{1}{\bar{F}_V}\right)^{\leftarrow}(v), \quad v > 0,$$

and the inverse is $g^{\leftarrow}(y) = \inf\{v : g(v) \ge y\}$ for a nondecreasing function g. Informally, letting $\mathcal{L}(v) \equiv 1$ in (12) we have

$$\bar{F}_V(v) = v^{-\alpha}, \quad b(v) = v^{1/\alpha}.$$

The slow growth condition is then given by (consistent with (17) for large T)

$$(\lambda(T)\cdot T)^{\frac{1}{\alpha}} \ll T \ \text{ or } \ \lambda(T) \ll T^{\alpha-1}, \tag{18}$$

and the fast growth condition by (consistent with (16) for large T)

$$T^{\alpha-1} \ll \lambda(T). \tag{19}$$

Convergence results for the arrival and departure processes. From [11], the convergence results with $\mathcal{L}(v) \equiv 1$ (used in Section II-B) for the arrival processes are:

1) Under the fast growth condition,

$$\frac{\lambda(T)}{\lambda(T)^{1/2}T^{(3-\alpha)/2}}\left(A^T(\cdot) - T\mu_{\text{on}}(\cdot)\right) \tag{20}$$

converges to FBM in $D[0,\infty)$ with the usual Skorokhod J_1-topology (see [13]).

2) Under the slow growth condition,

$$\frac{\lambda(T)}{b(\lambda(T)T)}\left(A_T(\cdot) - T\mu_{\text{on}}(\cdot)\right) \tag{21}$$

converges to SLM in finite dimensional distributions and in $D[0,\infty)$ with the M_1-topology (see [13]).

As in the model in Section I, for the departures we note that

$$\frac{1}{T^{1/2}}\int_0^{T(\cdot)}\left(\sum_j 1_{\{L(s)=j\}} - \pi(j)\right)\bar{p}(j)ds \tag{22}$$

converges to the Brownian motion $w(\cdot)$ in the Skorokhod J_1-topology.

B. Convergence theorems

Here we investigate the possible limit processes for the queue under fast growth and slow growth conditions. In each case the state space scalings lead to either the centered arrival or centered departure process having a nontrivial limit.

The scaling used and the limit results depend on the parameters of the arrival process $(\lambda(T), \alpha)$ and departure process (ν). Interestingly, the inequalities

$$\frac{\nu}{2} + \frac{3}{2} \lesseqgtr \alpha \tag{23}$$

are important for determining the scaling and limit processes in both the fast and slow growth regimes. In the fast growth regime, the limit process is RBM except for a case when the intensity $\lambda(T)$ lies in a region defined using (23) resulting in a RFBM limit model for the queue (Theorem 3 below). In the slow growth regime the limit process is RSLM, except for a case where (23) is used to define a region for $\lambda(T)$ where the limit process is RBM (Theorem 6 below).

Fast growth regime. In this regime, for large T, $T^{\alpha-1} \ll \lambda(T)$ by (16) and the limit models are determined by the inequalities (23) as shown next.

Theorem 1: Let

$$\frac{\nu}{2} + \frac{3}{2} < \alpha$$

and take

$$\beta(T) = \frac{T^{\nu}}{\sqrt{T^{1+\nu}}} = \frac{1}{T^{(1-\nu)/2}}. \tag{24}$$

Then x^T converge to RBM with a drift component given by (11).

Proof. Part 1: unreflected process.
From $\frac{\nu}{2} + \frac{3}{2} < \alpha$, it follows

$$T^{2-\alpha+\nu} \ll T^{\alpha-1} \ll \lambda(T).$$

With $\beta(T)$ in (24) we have that $M^{d,T}$ converges to BM—this follows from the techniques discussed in the appendix of [2]. There a *perturbed test function* method using a martingale characterization of Brownian motion was applied (using Theorems 7.4.4 and 10.8.1 in [8]).

Noting that for large T

$$\beta(T)\frac{\lambda(T)^{1/2}T^{(3-\alpha)/2}}{\lambda(T)} = \frac{T^{1-\alpha/2+\nu/2}}{\lambda(T)^{1/2}} \ll 1, \tag{25}$$

we have that

$$\begin{aligned} M^{a,T}(t) \quad &= \quad \frac{T^{1-\alpha/2+\nu/2}}{\lambda(T)^{1/2}} \\ &\quad \times \frac{\lambda(T)}{\lambda(T)^{1/2}T^{(3-\alpha)/2}}\left(A^T(t) - Tt\mu_{\text{on}}\right) \end{aligned} \tag{26}$$

converges to the zero process. Indeed, the first term in (26) converges to 0 from (25) and the second term in (26) converges to FBM from (20). From the second condition in (2), which with (14) gives the heavy traffic assumption, the $u^T(j,x)$ is such that the corresponding term in (15) (third line down) gives the drift rate $\sum_j \pi(j)\bar{\lambda}^d(j)u(x)t$ in the limit process.

Part 2: reflected process.

We wish to show $\{z^T\}$ converges to a reflection process, keeping the queue size nonnegative. A standard approach (see [6] or [7]) is to establish weak convergence to process z ($z^T \Rightarrow z$) and apply the Skorohod representation theorem to show z is a reflection process. A key for establishing the weak convergence is showing that the collection of processes $\{z^T; T\}$ is tight.

Following the discussion in [6], Section 5.2, an "asymptotic continuity" argument along with the the reflection map from the Skorohod problem can be used to show $\{z^T; T\}$ is tight and we outline the argument for our case next. From (15), define

$$\begin{aligned} h^T(t) \quad &\doteq \quad x^T(0) + M^{a,T}(t) - M^{d,T}(t) \\ &\quad -\beta(T)\int_0^{Tt}\sum_j 1_{\{L^T(s)=j\}}\bar{\lambda}^d(j)u^T(x(s))\,ds. \end{aligned}$$

Due to the convergence of $M^{d,T}$ to BM, $M^{a,T}$ to the zero processes, and the fact that $x^T(0)$, $\bar{\lambda}^d u^T(j,x)$ are bounded, the "asymptotic continuity" condition holds:

$$\lim_{\delta\to 0}\limsup_T P\left\{\sup_{t\le\Lambda}\sup_{s\le\delta}|h^T(t+s) - h^T(t)| \ge \mu\right\} = 0, \tag{27}$$

for each $\Lambda > 0$ and $\mu > 0$. This property implies that $\{z^T; T\}$ is tight by the following argument. The reflection term z^T satisfies the

$$z^T(t) = \max\left\{0, -\min_{s\le t} h^T(s)\right\}, \qquad (28)$$

where the right hand side is the reflection map (see [6]). From this we have,

$$z^T(\tau+t) - z^T(\tau) = \max\left\{0, -\min_{s\le t}\left[x^T(\tau) + h^T(\tau+s) - h^T(\tau)\right]\right\}. \qquad (29)$$

The tightness needed for weak convergence of z^T follows from (29) and the asymptotic continuity in (27) (see [6] for the details of the general criterion for showing tightness). ■

Remark 1: Apart from technicalities relating convergence of the various components of the unreflected process in different topologies (i.e., uniform, J_1, and M_1), an application of the continuous mapping theorem applies in a similar way for Theorems 2-6 and thus will not be repeated. It appears not possible at the current time to extend the continuous mapping theorem for all cases when u also had a queue-state dependence.

The following two theorems concern the opposite inequality in (23). In this case, the limit processes depend on a finer growth condition on $\lambda(T)$.

Theorem 2: Let

$$\frac{\nu}{2} + \frac{3}{2} > \alpha$$

and take

$$\beta(T) = \frac{T^\nu}{\sqrt{T^{1+\nu}}} = \frac{1}{T^{(1-\nu)/2}}.$$

Furthermore suppose that $\lambda(T)$ is such that for consistency

$$T^{\alpha-1} \ll T^{2-\alpha+\nu} \ll \lambda(T). \qquad (30)$$

Then x^T converges to RBM with a drift component given by (11).

Proof. The argument is the same as before where we only need to note that under (30) we still have that the first term in (26) converges to 0. ■

Theorem 3: Let

$$\frac{\nu}{2} + \frac{3}{2} > \alpha$$

and take

$$\beta(T) = \frac{\lambda(T)}{\lambda(T)^{1/2} T^{(3-\alpha)/2}}. \qquad (31)$$

Furthermore, suppose that $\lambda(T)$ is such that

$$T^{\alpha-1} \ll \lambda(T) \ll T^{2-\alpha+\nu}. \qquad (32)$$

Then x^T to converges to RFBM.

Proof. Using the $\beta(T)$ in (31) and noting (20), we have that $M^{a,T}$ converges to FBM. Note that

$$\beta(T)T^{(1-\nu)/2} = \frac{\lambda(T)^{1/2}}{T^{1-\alpha/2+\nu/2}} \ll 1. \qquad (33)$$

Then

$$M^{d,T}(t) = \beta(T)T^{(1-\nu)/2} \times \frac{1}{T^{(1+\nu)/2}} \int_0^{T^{1+\nu}t} \sum_j \left(1_{\{L(s)=j\}} - \pi(j)\right) \bar{\lambda}(j) p(j)\, ds \qquad (34)$$

converges to the zero process. Indeed, the first term in (34) converges to 0 from (33) and the second term in (34) converges to BM from (22). The asymptotic continuity approach for obtaining RFBM appears promising due to the continuity properties of the paths of fractional Brownian motion. ■

Slow growth regime. In this regime, for large T, $\lambda(T) \ll T^{\alpha-1}$. For Theorems 4 and 5 we note that the arrivals "dominate" departures and queue process converges to RSLM.

Theorem 4: Let

$$\frac{\nu}{2} + \frac{3}{2} > \alpha$$

and take

$$\beta(T) = \frac{\lambda(T)}{\lambda(T)^{1/\alpha} T^{1/\alpha}}. \qquad (35)$$

Then x^T converges to RSLM.

"Proof". From $\frac{\nu}{2} + \frac{3}{2} > \alpha$ it follows that $(\alpha-1)^2 < 1 - \frac{\alpha}{2} + \frac{\alpha\nu}{2}$ and we have

$$\lambda(T) \ll T^{\alpha-1} \ll T^{\frac{1-\alpha/2+\nu\alpha/2}{\alpha-1}}.$$

With the $\beta(T)$ in (35), we have from (21) that M_a^T converges to SLM. We note that

$$\beta(T)T^{(1-\nu)/2} = \frac{\lambda(T)^{1-1/\alpha}}{T^{1/\alpha+\nu/2-1/2}} = \left(\frac{\lambda(T)}{T^{\frac{1-\alpha/2+\nu\alpha/2}{\alpha-1}}}\right)^{\frac{\alpha-1}{\alpha}} \ll 1. \qquad (36)$$

From this it follows, as in the discussion of Theorem 3, that $M^{d,T}$ converges to the zero process. A remaining issue left to work out are the details for convergence of the reflection process since SLM is a discontinuous process; consequently, the asymptotic continuity approach described in the proof of Theorem 1 cannot be applied. ■

The following two theorems concern the opposite inequality in (23). As in the fast growth regime, the limit processes depend on a finer growth condition on $\lambda(T)$.

Theorem 5: Let

$$\frac{\nu}{2} + \frac{3}{2} < \alpha$$

and take

$$\beta(T) = \frac{\lambda(T)}{\lambda(T)^{1/\alpha} T^{1/\alpha}}. \qquad (37)$$

Furthermore, suppose that $\lambda(T)$ is such that

$$\lambda(T) \ll T^{\frac{1-\alpha/2+\nu\alpha/2}{\alpha-1}} << T^{\alpha-1}.$$

Then x^T converges to RSLM.

"Proof". In this case we have that (36) still holds and so the discussion in Theorem 4 carries over to this case. ■

Theorem 6: Let

$$\frac{\nu}{2} + \frac{3}{2} < \alpha$$

and take

$$\beta(T) = \frac{1}{T^{(1-\nu)/2}}. \quad (38)$$

Furthermore, suppose that $\lambda(T)$ is such that

$$T^{\frac{1-\alpha/2+\nu\alpha/2}{\alpha-1}} \ll \lambda(T) \ll T^{\alpha-1}.$$

Then x^T to converges to RBM.

Proof. Under the $\beta(T)$ in (38) and using (22), $M^{d,T}$ converges to Brownian motion. Note that

$$\beta(T)\frac{\lambda(T)^{1/\alpha}T^{1/\alpha}}{\lambda(T)} = \frac{T^{1/\alpha+\nu/2-1/2}}{\lambda(T)^{1-1/\alpha}} \ll 1, \quad (39)$$

so that by (21) we have that $M^{a,T}$ converges to the zero process. ■

Remark 2: We assumed for simplicity in Theorems 1-6 that the slowly varying function appearing in (12) is $\mathcal{L} \equiv 1$. However, the conditions determining the various limit processes can also be formulated for arbitrarily slowly varying functions $\mathcal{L}(v)$ (the most general case). For example, as shown in [11], fast growth is equivalent to

$$T^{\alpha-1} \ll \lambda(T) \cdot \mathcal{L}(T).$$

Then one can show that Theorems 1-3 continue to hold if (30) is replaced by

$$T^{\alpha-1} \ll T^{2-\alpha+\nu}\mathcal{L}(T)^2 \ll \lambda(T) \cdot \mathcal{L}(T)$$

and (32) is replaced by

$$T^{\alpha-1} \ll \lambda(T) \cdot \mathcal{L}(T) \ll T^{2-\alpha+\nu}\mathcal{L}(T)^2.$$

Remark 3: If

$$\frac{\nu}{2} + \frac{3}{2} = \alpha,$$

then, loosely speaking, the limit process is expected to be a reflection of the "combinations" of Brownian motion, stable Lévy motion, or fractional Brownian motion.

Remark 4: The queue state dependence in $u^T(j,x)$ precludes (at the current time) obtaining the limit models using an alternative approach where the continuous mapping theorem is applied to the map in (28). However, for only channel state dependence, i.e. $u^T(j)$, this map is continuous in appropriate topologies for all the cases considered here and so is a promising approach.

III. Summary

Figure 1 summarizes Theorems 1 - 6. Note that the inequalities

$$\frac{\nu}{2} + \frac{3}{2} \lesseqgtr \alpha$$

relating the channel and arrival process are used in the fast growth and slow growth regimes. They define regions for

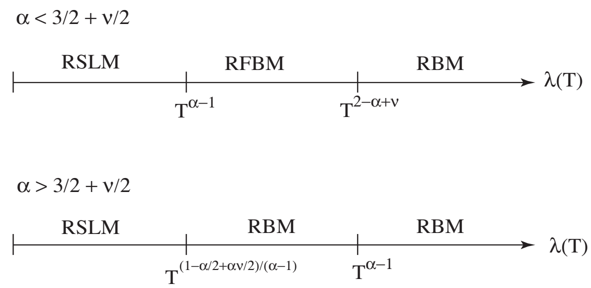

Fig. 1. Summary of possible limit models.

the "growth" conditions on $\lambda(T)$ which cover the various possible limit models for the queue dynamics.

To the best of our knowledge, these are the first results of their kind for wireless systems and have no analog in the current work in wireline models. Obtaining the queue models incorporating LRD/HT characteristics is an important step in characterizing the queue behavior in high-speed wireless systems. A natural next step is to consider the resource allocation problem in networks with competing queues—our future work will broadly be in this direction.

References

[1] J. Beran, R. Sherman, M.S. Taqqu, and W. Willinger, *Long-range dependence in variable-bit-rate video traffic,* " IEEE Transactions on Communication, **43**(1995), pp. 1566-1579.

[2] R.T. Buche and H.J. Kushner, *Control of mobile communications with time-varying channels in heavy traffic*, IEEE Trans. Automatic Control, **47** (2002), pp. 992-1003.

[3] K. Dębicki, M. Mandjes, *Traffic with an fBm Limit: Convergence of the Stationary Workload Process*, Queueing Systems, **46** (2004), pp. 113-127.

[4] R. Delgado *A reflected fBm limit for fluid models with On/Off sources under heavy traffic*, Stochastic processes and their applications (2006), (in press) doi:10.1016/j.spa.2006.07.001.

[5] T. Konstantopoulos, S. Lin *Fractional Brownian Approximations of Queueing Networks*, Stochastic Networks, Lecture Notes in Statistics, **117**, Springer, NY, 1996.

[6] H.J. Kushner, *Heavy Traffic Analysis of Controlled Queueing and Communication Networks.* Springer, New York, 2001.

[7] H.J. Kushner, P. Dupuis, *Numerical Methods for Stochastic Control Problems in Continuous Time, Second Edition*, Springer, New York, 2001.

[8] H.J. Kushner *Stochastic Approximation Algorithms and Applications.* Springer, NY 1997.

[9] A. Krendzel, Y. Koucheryavy, J. Harju, S. Lopatin, *Network Planning Problems in 3G/4G Wireless Systems"*, The 1st COST 290 Management Committee Meeting, Malta, Oct. 2004.

[10] W.E. Leland, M.S. Taqqu, W. Willinger, and D.V. Wilson, *On the self-similar nature of Ethernet traffic*, IEEE/ACM Transactions on Networking, **2**:1 (1994), pp. 1-15.

[11] T. Mikosch, S. Resnick, H. Rootzén, *Is Network Traffic Approximated by Stable Lévy Motion or Fractional Brownian Motion?* The Annals of Applied Probability, **17**, No. 1, pp. 23-68.

[12] A. L. Stolyar, *Max Weight scheduling in a generalized switch: State space collapse and workload minimization in heavy traffic"*, The Annals of Applied Probability, **14**(1): 1–53 February 2004.

[13] W. Whitt, *Stochastic Process Limits* Springer, New York, 2002.