Correction to Thm. 2
on 03/19 lecture
(working towards optional stopping thm)

In the proof of Thm 2:

We had

$$\int_{A\cap\{T\le k\}} X_T\,dP = \int_{A\cap\{T\le k\}} X_{T\wedge k}\,dP = \int_{A\cap\{T\le k\}} X_k\,dP \overset{\text{u.i.}}{=} \int_{A\cap\{T\le k\}} X_\infty\,dP$$

can see this by (A) in Thm 1 taking $S = T\wedge k$ and noting $A\cap\{T\le k\} \in \mathcal{F}_S$

Then we have

$$\int_{A\cap\{T\le k\}} X_T\,dP = \int_{A\cap\{T\le k\}} X_k\,dP = \int_{A\cap\{T\le k\}} X_\infty\,dP$$

$\boxed{\text{We need to insert:}}$

insert

By Dominated convergence (by u.i. $\sup_n E|X_n| \le c$, so let bounding r.v. be C)

$$\int_{A\cap\{T\le\infty\}} X_T\,dP = \int_{A\cap\{T\le\infty\}} X_\infty\,dP. \text{ Note}$$

$A\cap\{T\le\infty\} \in \mathcal{F}_T$ so we have $X_T = E[X_\infty \mid \mathcal{F}_T]$ as in notes
$(=A)$