MA/ST 747, Spring 2009
Homework 4
Due: In-class, Tuesday, April 14

1. In class, we stated that for T a stopping time

$$\mathcal{F}_T \triangleq \{A \in \mathcal{F}_\infty : A \cap \{T \leq n\} \in \mathcal{F}_n, \text{ all n}\}.$$

   (a) Show that an equivalent definition is

$$\mathcal{F}_T \triangleq \{A \in \mathcal{F}_\infty : A \cap \{T = n\} \in \mathcal{F}_n, \text{ all n}\}.$$

   (b) Show that for $A \triangleq \{T = n\}$, $A \in \mathcal{F}_T$.

2. Durrett, Chapter 3, Exercise 1.6.

3. Let $(X_n)_{n\geq 1}$ be a adapted to $(\mathcal{F}_n)_{n\geq 1}$ and $(B_n)_{n\geq 1}$ be a sequence of Borel sets in $I\!R$. Let

$$T \triangleq \inf\{n \geq 1 : X_n \in B_n\}.$$

   Finally, define $X_T \in \bar{I\!R}$ (extended reals) by

$$X_T = \begin{cases} X_m & \text{if } T = m \\ \limsup_{n\to\infty} X_n & \text{if } T = \infty. \end{cases}$$

   (a) Show that T is a stopping time with respect to $(\mathcal{F}_n)$.

   (b) Show that X_T is $\mathcal{F}_T$-measurable.

4. Durrett, Chapter 4, Exercise 7.6. *Note:* $S_n = \xi_1 + \xi_2 + ... + \xi_n$, where the (ξ_i) are i.i.d.

5. Durrett, Chapter 4, Exercise 7.7. *Note:* Again, the (ξ_i) are i.i.d. *Hint:* Consider $Ee^{\theta\xi_1}$ and apply the result (adjusted for this problem) of problem 4.

6. Durrett, Chapter 5, Exercise 1.4.

7. Durrett, Chapter 5, Exercise 1.6.

8. Durrett, Chapter 5, Exercise 1.7.

9. Durrett, Chapter 5, Exercise 1.8. *Hint:* The probabilities needed to compute the conditional probability in (i) have the form

$$\int_0^1 x^m (1-x)^k \, dx = \frac{m!k!}{(m+k+1)!}$$