MA/ST 747, Spring 2009
Homework 1
Due: In-class, Thursday, Jan. 29

1. In class we saw/will see that for proving

$$\sum_{n=1}^{\infty} P(|X| \geq n) \leq E|X| \leq 1 + \sum_{n=1}^{\infty} P(|X| \geq n),$$

we got to the point where the above was shown, except with the expression

$$\sum_{n=0}^{\infty} nP(\Lambda_n), \quad \Lambda_n = \{\omega : n \leq |X(\omega)| < n+1\}$$

used in place of

$$\sum_{n=1}^{\infty} P(|X| \geq n).$$

Show that

$$\sum_{n=0}^{\infty} nP(\Lambda_n) = \sum_{n=1}^{\infty} P(|X| \geq n)$$

to finish the proof.

Hint: Consider $\sum^N$ and then take $N \to \infty$ for two cases: the case where X is integrable and then not integrable.

2. Durrett, Ch.1, Exercise 3.1

3. Durrett, Ch.1, Exercise 3.3.

Hint; Use the fact (in (5.1), Appendix) that you can bound below $\phi(x)$ by $l(x)$, a linear function, where $\phi(EX) = l(EX)$.

4. Durrett, Ch.1, Exercise 3.8

5. Durrett, Ch.1, Exercise 3.14

6. Durrett, Ch.1, Exercise 3.15 (let indexing start at $n = 1$)

Hint: Let $Y_n = X_n + X_1^-$, and apply the Monotone Convergence Theorem.

7. Durrett, Ch. 1, Exercise 3.18

Hints: You may want to use Exercise 3.17 results. You should prove the absolute summability of the series; hint for doing this: consider Jensen's inequality with $\psi(\cdot) = |\cdot|$ and the given integrability result for X.