MA 573 — PROJECT 3

Due: Wednesday, October 24

(1) Consider the unforced equation

$$\begin{aligned} \frac{\partial u}{\partial t} &= \alpha \frac{\partial^2 u}{\partial x^2} \\ u(t,0) &= u(t,2) = 0 \\ u(0,x) &= \sin(\pi x/2) \end{aligned} \tag{1}$$

with $\alpha = 0.7$. You should determine the analytic solution and then approximate the solution to (1) using central differences in space and backward differences in time. In the same figure, plot the true and approximate solutions at $T = 1$ for appropriate stepsizes to indicate the convergence of the method. You should also compute the maximum absolute error over your space-time grid and report that for a representative set of stepsizes. Is your method achieving the prescribed convergence rate?

(2) Now consider the forced equation

$$\begin{aligned} \frac{\partial u}{\partial t} &= \alpha \frac{\partial^2 u}{\partial x^2} + x(x-2)\sin(3\pi t) \\ u(t,0) &= u(t,2) = 0 \\ u(0,x) &= \sin(\pi x/2) \end{aligned} \tag{2}$$

with $\alpha = 0.7$. Discuss the convergence of the method and indicate techniques that you are using to ascertain convergence (you do not need to compute an analytic solution for this case). When does the solution appear to have reached steady state behavior?

Hint: You can use the `diag` command to create the tridiagonal, Toeplitz matrix for your finite difference discretization.