North Carolina State University | Department of Mathematics

MA 573 (Fall 2006) Project #2 September 19, 2007

Due Date: September 26, 2007

In this exercise you will attempt to use the harmonic oscillator (mass-spring-dashpot) model of Project #1 to describe vibrational data from a beam. You will take *accelerometer* data from a vibrating beam in the CRSC/Math Instructional and Research Laboratory. The mathematical model is given by

$$m\frac{d^2y(t)}{dt^2} + c\frac{dy(t)}{dt} + ky(t) = 0$$

with initial conditions

$$y(t_0) = y_0, \qquad \frac{dy(t_0)}{dt} = v_0,$$

or

$$\frac{d^2y(t)}{dt^2} + C\frac{dy(t)}{dt} + Ky(t) = 0$$

with initial conditions

$$y(t_0) = y_0, \qquad \frac{dy(t_0)}{dt} = v_0.$$

The above two models are equivalent when $C = c/m$ and $K = k/m$ if $m \neq 0$.

1.) Collect data:
Excite the beam with a sinusoidal input at the first fundamental frequency of the beam (approximately 11 Hz). Use the piezoceramic patches for the exciting actuator. Terminate your exciting input to the beam at $t = t_0$ when $\frac{d}{dt}y(t_0) = 0$ (see Figure 1) and measure $\hat{y}_0 = y(t_0)$ (so y_0 and and v_0 are assumed to be given by observations.)

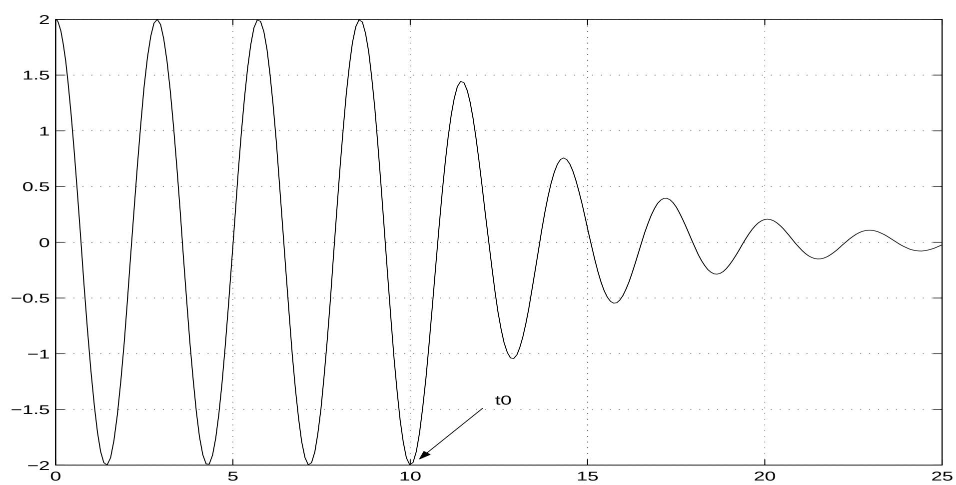

Figure 1: Beam Excitation

Take data on $[t_0, t_1]$: $\hat{a}_i^d$ which are observations for $\ddot{y}_{\text{mod}}(t_i; C, K)$.

2.) Formulate and carry out the corresponding inverse problem for $q = (C, K)$ with

$$J_n(q) = \frac{1}{n} \sum_{i=1}^{n} |\hat{a}_i^d - \ddot{y}_{\text{mod}}(t_i; C, K)|^2$$

(a) Estimate $q^* = (C^*, K^*)$ from the data, obtaining $q_n^* = (C_n^*, K_n^*)$ so that $J_n(q_n^*)$ is the residual.

(b) Estimate $q^{**} = (0, K^{**})$ - the undamped model - obtaining $q_n^{**} = (0, K_n^{**})$ so that $J_n(q_n^{**})$ is the residual.

(c) Use the $\chi^2(1)$ test to test for the significance of your improved fit to the data by allowing nontrivial damping $C \neq 0$ in the model.

3.) Repeat 1.) and 2.) above by exciting the beam with a sinusoidal input at the second fundamental frequency of the beam (approximately 60 Hz).